

was used in an attempt to satisfy the uniform variance assumption of the ANOVA model. The difference

$$t = \sin^{-1}\sqrt{\hat{X}} - \sin^{-1}\sqrt{X} \quad (5-2)$$

was the response variable to quantify errors in proportion estimates.

5.1.2 ANOVA MODEL

The experimental design is a three-way classification with the following model:

$$t_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \gamma_k + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + e_{ijk} \quad (5-3)$$

where

μ = Mean response

α_i = Effect of *i*th site

β_j = Effect of *j*th biophase

$(\alpha\beta)_{ij}$ = Interaction between *i*th site and *j*th biophase

γ_k = Effect of *k*th AI

$(\alpha\gamma)_{ik}$ = Interaction between *i*th site and *k*th AI

$(\beta\gamma)_{jk}$ = Interaction between *j*th biophase and *k*th AI

$(\alpha\beta\gamma)_{ijk}$ = Three-way interaction between *i*th site, *j*th biophase, and *k*th AI

and e_{ijk} is the random error component. It is assumed that $(\alpha\beta\gamma)_{ijk} \equiv 0$ and e_{ijk} is independent and identically distributed as normal with mean 0 and variance σ_e^2 . The model is a mixed one

in which biophase and AI are considered "fixed" effects and site a random effect. The two sites are considered to constitute a random sample from a large population of sites.

The objectives of this experimental study can now be stated in terms of testing the following hypotheses:

- No "main" effect due to
 - a. site
 - b. biophase
 - c. AI
- No interaction between
 - d. site and biophase
 - e. site and AI
 - f. biophase and AI

5.1.3 RESULTS AND CONCLUSIONS

An examination of data in table 5-1 indicates that proportion estimates varied considerably more in biophase 1 than in other biophases for segment 1969 but not for segment 1976. This suggests that it may be inappropriate to assume the error variance component to be the same for all combinations of sites and biophases or of sites, biophases, and AI's. To explore this conjecture further, analyses of variance were carried out both with and without biophase 1 data. The numerical results obtained for the ANOVA performed on all 112 data points are given in table 5-3(a). Because there was no replication of the data, an unbiased estimate of the error variance could not be obtained; only one observation was available for each combination of factors. The residual mean square error provided an unbiased estimate of the error variance and the three-way interaction (ITS/biophase/AI) variance component.

TABLE 5-3.- ANALYSES OF VARIANCE OF INTENSIVE TEST SITE DATA

(a) With biophase as a factor

Source of variation	Degrees of freedom	Sum of squares	Mean square error	F-ratio
Site	1	0.11113	0.11113	4.21
Biophase	3	.02419	.00806	.11
AI	13	.70676	.05437	1.10
ITS vs biophase	3	.22339	.07446	^a 2.82
ITS vs AI	13	.64351	.04950	^a 1.87
Biophase vs AI	39	.91976	.02358	.89
Residual (site vs biophase vs AI)	39	1.03020	.02642	
Total	112	3.65894		

(b) Without biophase 1

Source of variation	Degrees of freedom	Sum of squares	Mean square error	F-ratio
Site	1	0.26860	0.26880	^b 13.64
Biophase	2	.01933	.00967	1.54
AI	13	.40112	.03086	.74
ITS vs biophase	2	.01259	.00629	.32
ITS vs AI	13	.54343	.04180	^a 2.12
Biophase vs AI	26	.34931	.01344	.68
Residual (site vs biophase vs AI)	26	.51247	.01971	
Total	83	2.01685		

(c) With biophase treated as a replicate

Source of variation	Degrees of freedom	Sum of squares	Mean square error	F-ratio
Site	1	0.26860	0.26880	^b 16.8
AI	13	.40112	.03086	.73
Site vs AI	13	.54343	.04180	^a 2.61
Error	56	.89370	.01596	

^aSignificant at the 5-percent level.

^bSignificant at the 1-percent level.

Since the latter was assumed to be zero, the residual mean square error became an unbiased estimate of the error variance. On this basis, when F-tests were applied at the 5-percent level of significance, the following conclusion was reached: There was a significant interaction between ITS and AI, and between ITS and biophase, but no significant interaction between biophase and AI. Because of the significant interactions, one cannot arrive at any definitive conclusion about the significance of the individual factors of site, AI, and biophase.

Data investigation suggested that biophase 1 was causing the interaction between ITS and biophase. On the average, proportions were underestimated in biophase 1 and overestimated in biophases 2, 3, and 4 for segment 1969 but the reverse was the case for segment 1976. The data also revealed a lack of homogeneity between biophase 1 and other biophases, and this may be the cause of some of the interaction.

When biophase 1 was omitted in the data analysis, the results of the ANOVA were as listed in table 5-3(b). The F-test was applied on the same basis as for the (a) portion of the table and the following results were obtained:

- a. There was significant interaction between ITS and AI.
- b. There was no significant interaction between ITS and biophase.
- c. The site effect was highly significant.
- d. There was no significant interaction between AI and biophase.
- e. The biophase effect was not significant.

Since biophase was not a significant factor in terms of its main effect or its interaction with other factors, it could be "replicated"; i.e., sums of squares involving biophase terms could be pooled to form a more precise estimate of error variance, and

thus a better evaluation of other factors could be made. Data for table 5-3(c) were obtained by pooling the sums of squares due to biophase, ITS \times biophase, and AI \times ITS \times biophase in table 5-3(b). Once again the same conclusion was reached; i.e., there was significant interaction between ITS and AI, and the ITS effect was highly significant. Averaging over sites, no significant differences between AI's were found, but this finding has little significance since it was already seen that AI's performed inconsistently between the two sites; i.e., the AI \times site interaction was significant.

Based on the above analysis, it was concluded that:

- a. The CAMS error in proportion estimation varied significantly from one ITS to another.
- b. There was significant difference in the relative performance between AI's from one segment to another.
- c. Biophase 1 caused interaction between ITS and biophase. If the two ITS's were not a random sample from a larger population, inference about the site factor could not be widely applied.

5.2 FOUR-AI STUDY OF THE EFFECT OF SMALL GRAINS PROPORTION, AMOUNT OF TRAINING DATA, AND BIOPHASE

In this experiment, four AI's, working independently and using the CAMS rework procedures, analyzed all of the acquisitions over the 23 Phase I ITS's listed in appendix C which have acquisitions satisfying the CAMS rework criteria. The results were used to study (1) the effect of the proportion of small grains in the segment on proportion error (section 5.2.1), (2) the effect of the amount of training data on proportion error (section 5.2.2), and (3) the effect of biophase on labeling accuracy (section 5.2.3).

5.2.1 EFFECT OF THE PROPORTION OF SMALL GRAINS IN THE SEGMENT

Figure 5-1 is a plot of proportion error as a function of ground truth small grains proportions. Proportion error is defined as

$$\hat{X} - X$$

where

\hat{X} = CAMS estimated small grains proportions

X = Ground-observed small grains proportions.

The plot shows that the sites that were low in small grains were mostly overestimated and the sites that were high in small grains were mostly underestimated. The same type of plot was made for each biophase, each AI, and each group of ITS's within a state. All plots reflected the same behavior as that depicted in figure 5-1. This behavior can be explained theoretically as follows: Let X be the proportion of small grains in a segment and \hat{X} its estimate made by CAMS. Then, the expected proportion error (i.e., bias) can be expressed as

$$\begin{aligned} E(\hat{X}) - X &= X(1 - \alpha) + (1 - X) \beta - X & (5-4) \\ &= \beta - (\alpha + \beta)X \end{aligned}$$

where α denotes the proportion of small grains pixels classified as "other" (i.e., non-small-grains) and β is the expected proportion of "other" pixels classified as small grains. So, for a fixed value of $(\alpha + \beta)$, the bias in \hat{X} is a decreasing function of X. Moreover, if $X \leq 1/2$,

$$\begin{aligned} E(\hat{X}) - X &\geq (\beta - \alpha)/2 & (5-5) \\ &\geq 0, \text{ provided } \beta \geq \alpha \end{aligned}$$

and if $X > 1/2$,

$$\begin{aligned} E(\hat{X}) - X &< (\beta - \alpha)/2 & (5-6) \\ &< 0, \text{ provided } \beta < \alpha \end{aligned}$$

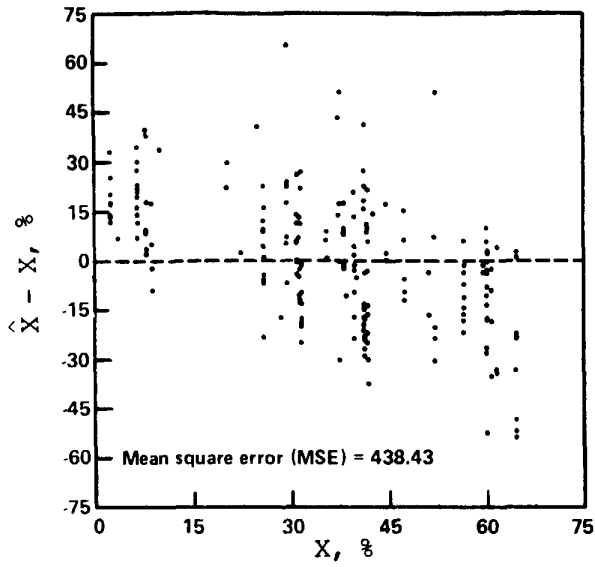


Figure 5-1.— Proportion error versus ground truth small grains proportions.

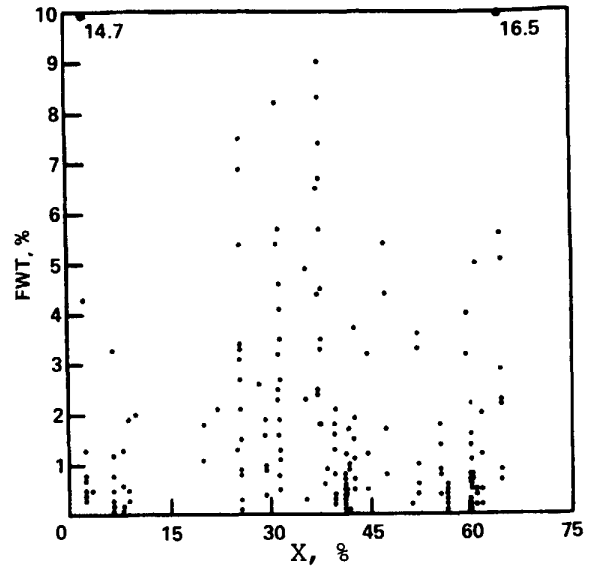


Figure 5-2.— Fraction of the classified wheat thresholded versus ground truth small grains proportions.

Data depicted in figure 5-1 seems to suggest that the conditions in equations (5-5) and (5-6) regarding the two types of errors are "fairly" well satisfied when X is very small or $X \geq 1/2$.

Thresholding

For a further explanation of these two types of errors, and thus dependence of proportion error on X , the thresholding aspect of the CAMS operation was investigated. (See page xvii for a definition of thresholding.) Since thresholded pixels were considered as "other", it was likely that fewer pixels classified as small grains would be thresholded from sites that had low small grains density; whereas, more pixels classified as small grains would be thresholded in sites with high small grains density. To determine whether thresholding could be a factor contributing to the trend depicted in figure 5-1, the fraction of the ground truth area which was actually small grains but was thresholded out (FWT) was plotted versus the ground truth small grains proportion (figure 5-2). The ground truth area is the portion of a segment for which ground truth was collected. FWT is the difference between a proportion estimate with no threshold and a proportion estimate with a 1-percent threshold. Data in figure 5-2 show no trend in FWT when plotted against X ; thus, thresholding can probably be discarded as an explanation of the results depicted in figure 5-1.

5.2.2 EFFECT OF THE AMOUNT OF TRAINING DATA

Since each of the four AI's worked independently, there were four different sets of training data for each ITS/biophase combination, each having a different number of pixels. Figure 5-3 shows a plot of proportion error versus the number of training pixels. Although one can see a slight reduction in proportion error as the number of training pixels increased, only a limited amount of information can be gained by the study of this plot, the reason being that the amount of training data selected by the AI's was very much site dependent. That is, the four AI's tended to choose only slightly

different amounts of training data within a given site, but the amount varied considerably from one ITS to another, since proportion error was found to be highly dependent on site. Figure 5-3 reflects mainly the differences in sites but does not reveal much about the effect of the number of training pixels.

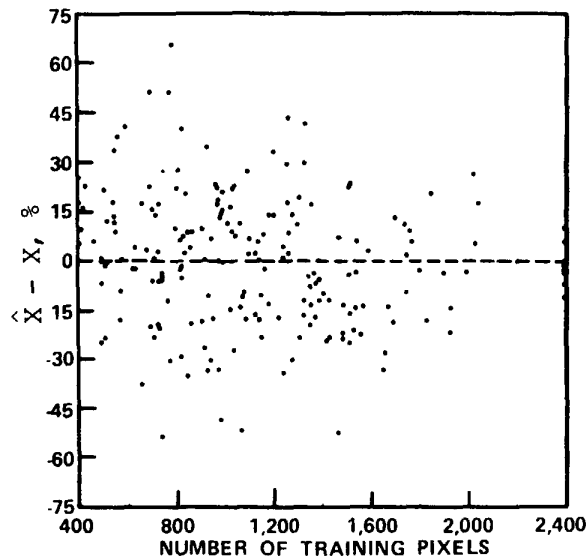
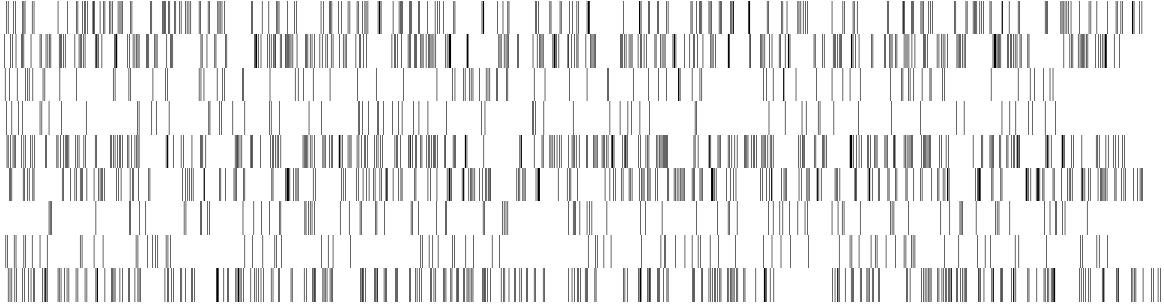
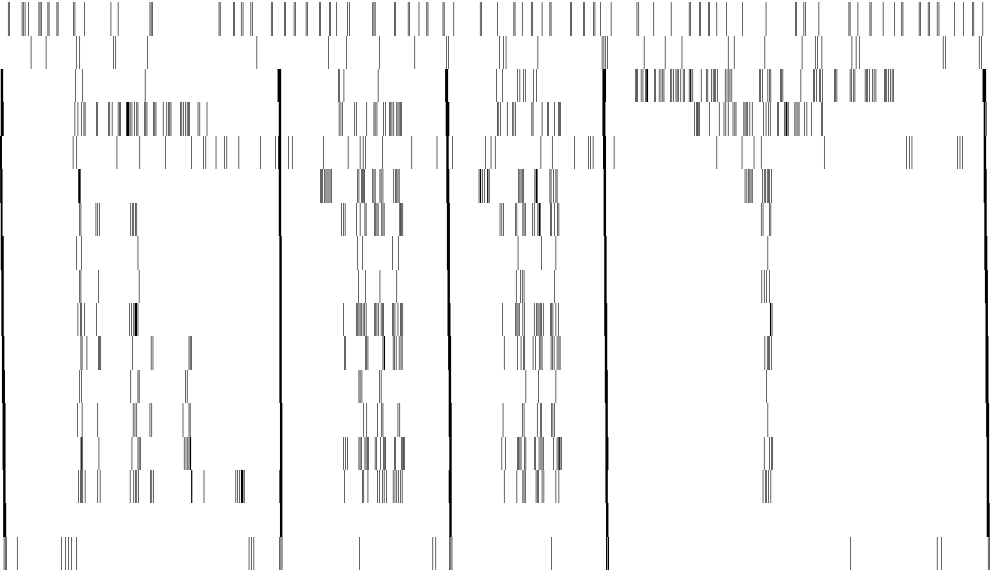


Figure 5-3.— Proportion error versus the number of training pixels.

5.2.3 EFFECT OF BIOPHASE ON LABELING ACCURACY

An effort was made to determine which biophase, or combination of biophases, provided the most success in labeling training fields. The area of ground truth varied from one ITS to another, whereas the AI-selected training fields were taken from any place within the segment. The accuracy data presented in table 5-4 refer only to those fields which were selected from the ground truth area of each segment.

The labeling accuracies varied a great deal from ITS to ITS but were relatively consistent for fields within sites. Thus, the tabulated results, which were based on two or more sites, were not very accurate as measures of average expected performance.



were selected to augment the knowledge acquired from the blind site study of the mixed and spring wheat sites in the USNGP.

The acquisition dates were selected to be representative of imagery available in actual operations. No more than one acquisition per biophase was used, and biophases were determined by actual crop calendars. All sites were ITS's over which at least two passes had been made, and each had an acquisition from either biophase 2 or 3 (table 5-5).

The sites were worked by each of four AI/Data Processing Analyst (AI/DPA) Teams randomly selected from teams which were familiar with CAMS rework methodology. Each AI/DPA Team reviewed the initial processing of each segment and accepted or reworked it for an estimate of the proportion of small grains in the segment.

5.3.1 COMPARISON OF CAMS REGULAR VERSUS CAMS REWORK RESULTS

Table 5-6 shows the results of the comparison of CAMS regular versus CAMS rework results. In 27 percent of the cases (12 out of 44), the results were improved by the CAMS rework procedure; in 23 percent of the cases (10 out of 44), the results were made worse by the CAMS rework procedure. In the other cases the segment was either declared unworkable or the original result was accepted. These results did not give any clear indication of whether or not the CAMS rework procedure gives better results than the CAMS regular procedure.

5.4 BLIND SITE PROPORTION ERRORS IN CAMS REGULAR AND REWORK PROCEDURES

Ground truth was collected from North Dakota and Montana LACIE operational segments which had been acquired and processed for at least two biophases. These sites were selected after biophase 2, thus providing a greater proportion of three and four acquisitions from a segment and allowing multitemporal processing. Aircraft

TABLE 5-5.- ACQUISITIONS FOR CAMS REWORK EXPERIMENT

Segment	Acquisition number for biophase			
	1	2	3	4
1687	74133		75205	
1960	74291		75150	
1962	74324	75131		
1963	74289	75131		
1965	75155	75191		
^a 1967				
1969	75161	75179	75215	75233
1970	75142	75179		75233
1978	74291		75133	
1979	74291		75133	
1980	74291		75133	
1986	75150	75169	75187	

^aNot suitable for processing because of lack of ground truth.

TABLE 5-6.- COMPARISON OF CAMS REGULAR VERSUS REWORK RESULTS

I = Improved results
W = Worse than original
N = Original accepted
U = Segment declared unworkable

Segment	AI/DPA Team			
	A	B	C	D
1687	I	W	I	U
1960	N	N	N	N
1962	I	I	N	W
1963	I	I	N	W
1965	N	N	W	N
1969	N	I	W	I
1970	N	W	W	W
1978	N	N	N	I
1979	N	N	N	N
1980	N	W	I	W
1986	I	I	U	U
Totals	12 I's	3 U's	10 W's	19 N's

photography was obtained for each of the 25 segments and photo-interpreted to obtain ground truth small grain proportions. (For some representative segments this ground truth was corroborated by visual inspection on the ground.)

Small grain proportion estimates obtained for these segments with CAMS regular and rework procedures were compared with their ground truth proportions. The CAMS regular estimates were those obtained using the regular CAMS operational procedures applied to the last acquisition available for each blind site. The CAMS reworked estimates were obtained for 19 segments. Of these, 10 were actually reprocessed and for the other nine segments, the original classification was declared acceptable by the rework team. This acceptance qualifies a segment to be considered a "reworked" segment.

Figures 5-4 and 5-5 show the CAMS proportion errors plotted as a function of the ground truth proportions. These figures appear to show that proportions were overestimated by the CAMS regular procedure and underestimated by the CAMS rework procedure; however, in both cases, the Wilcoxon matched-pairs signed-rank test* failed to reject the hypothesis of symmetric proportion errors around zero.

*R. P. Runyon and A. Haber, Fundamentals of Behavioral Statistics, Addison-Wesley Publishing Co., Reading, Mass., 1971, pp 263-265, 308, etc.

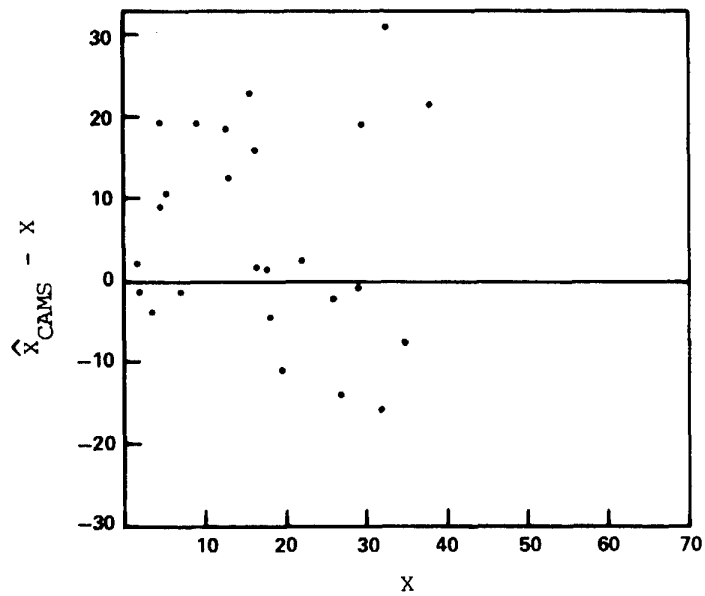


Figure 5-4.- Errors in the CAMS regular estimates as a function of X.

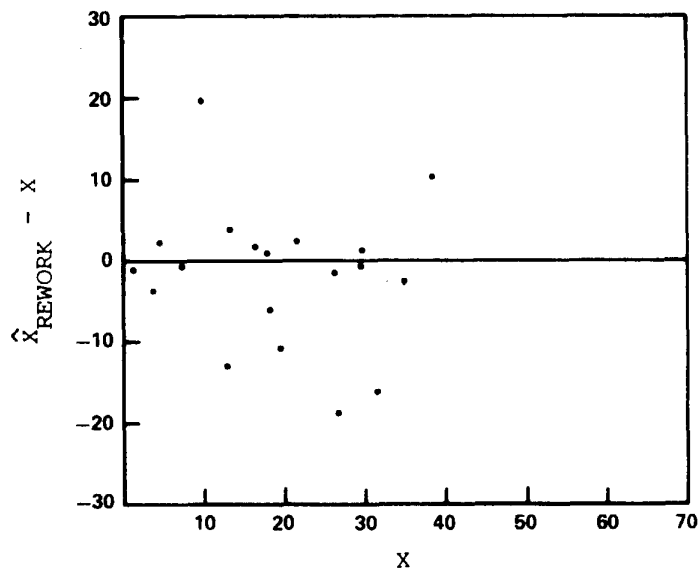


Figure 5-5.- Errors in the CAMS reworked estimates as a function of X.

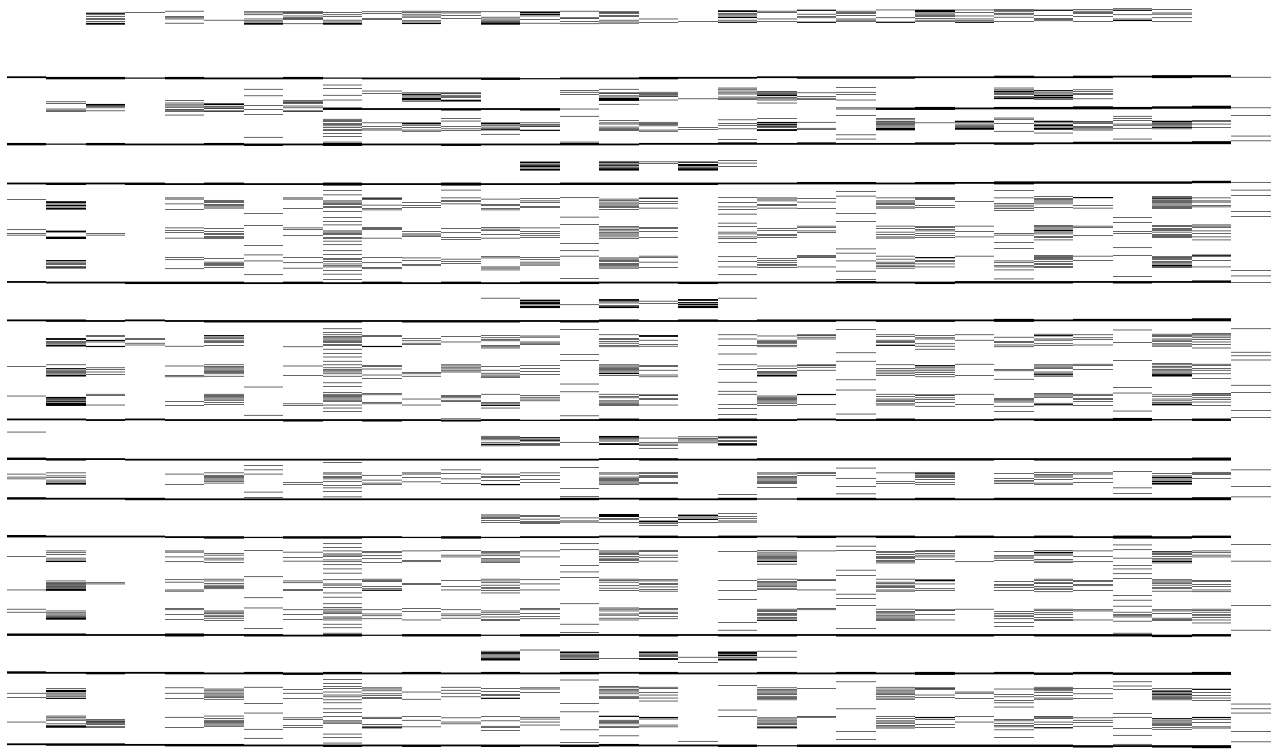
5.5 CROP CALENDAR VERIFICATION

To assess the performance of the adjustable crop calendar (ACC) the ACC output for the USGP region CRD's in which the Phase I ITS's were located was compared to average crop calendar output and to ground truth. The ACC for each ITS used in comparison is listed in table 5-7. Because ground-truth data were not received by the Data Acquisition, Preprocessing, and Transmission Subsystem (DAPTS) of the LACIE, data sets for the following ITS's were not analyzed and thus were not included in this study.

- Segment 1964, CRD 50, Ellis County, Kansas
- Segment 1962, CRD 50, Saline County, Kansas
- Segment 1968, CRD 20, Glacier County, Montana
- Segments 1687 and 1986, CRD 50, Hand County, South Dakota
- Segment 1967, CRD 10, Divide County, North Dakota

The Phase I biophases and their respective biological wheat stages are as follows:

Biophase	Biological wheat stage	
	Number	Activity
1	1	Planting
	2	Emergence
2	3	Jointing
3	4	Heading
4	5	Soft dough
	6	Ripening
	7	Harvest



—
—
—

The crop calendar comparisons are graphically depicted and discussed in the following subsections.

5.5.1 KANSAS (WINTER WHEAT)

Segment 1960, Finney County

Finney County is located in the north-central portion of the CRD. The wide range between the ACC and the ground-truth curves is attributed to differences in jointing dates between the ITS and USDA/SRS state averages (fig. 5-6). The jointing data on which the ACC was started was May 6, 1975. This date was supplied by the USDA/SRS office in Kansas and represents the CRD average 50-percent jointing date. In comparison, the ITS 50-percent jointing date was April 20, 1975.

Segment 1961, Morton County

Located in the extreme southwest corner of the CRD, the data from this ITS may not be representative of the entire CRD. However, the meteorological data used to effect the calendar adjustments were derived from stations located in Dodge City, Kansas, and Gage, Oklahoma. Dodge City, which is located in the extreme northeast corner of CRD 7, and Gage are equidistant from the ITS. An apparent discrepancy exists in the ground-truth data, inasmuch as the period between jointing and heading is too short to be realistic (fig. 5-6). If the dates for the other two ITS's are used as a guide, it would suggest that the jointing date is incorrect.

Segment 1963, Rice County

The location of this ITS is in the south-central part of the CRD. The ground-truth data do not compare favorably, especially in the early stages of development (fig. 5-6). The NOAA *Weekly Weather and Crop Bulletin* reported wheat development noticeably behind the normal curve on April 22, 1975. The state averages for Kansas

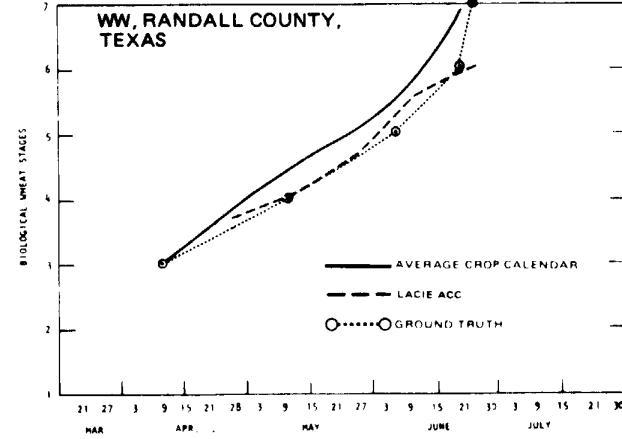
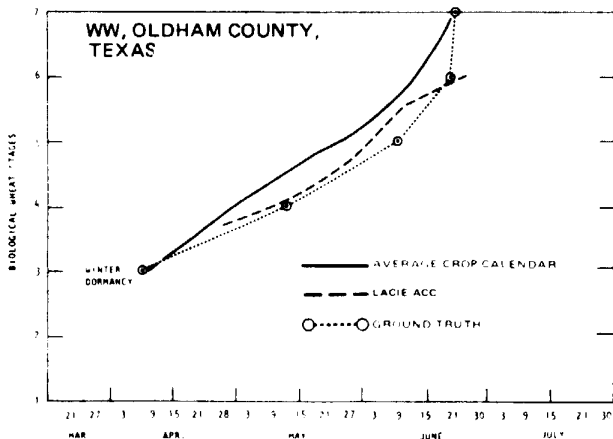
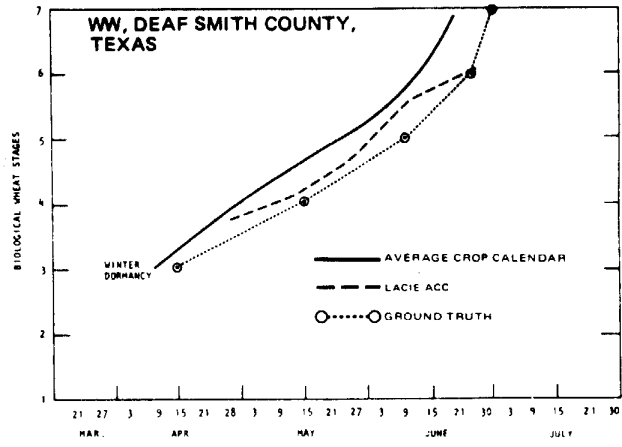
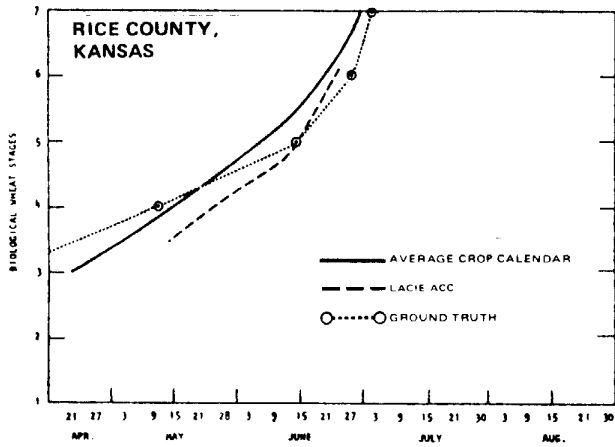
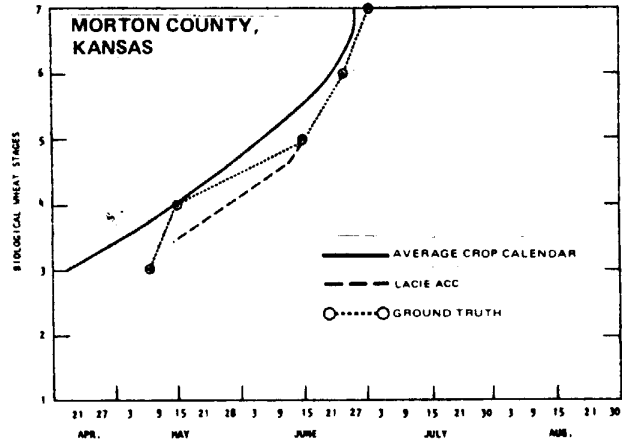
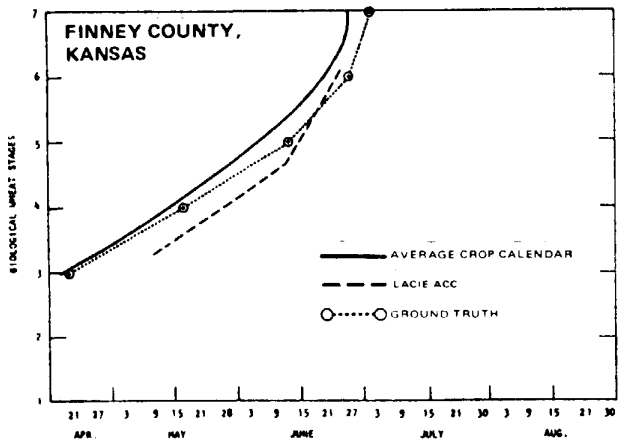


Figure 5-6.— Crop calendar comparisons (winter wheat).

reported 10 percent jointed compared to 45 percent in 1974 and a 40-percent average. The ITS ground-truth data reported 50 percent jointing on April 5. The state average reported the 50-percent jointing date as May 1. The 50-percent jointing date for the CRD, as supplied by the USDA/SRS, is May 3. The ground-truth date for 50-percent jointing is April 5. This, again, is the obvious contributor to the wide range between the ACC and ground truth from the jointing through the soft-dough stages. From all appearances, the ITS dates appear to be either (1) erroneous or (2) the development of wheat within the ITS for the 1975 season was a clear exception from the normal reported state and CRD averages.

The trend in all three of the comparisons for Kansas indicates a difference in the interpretation of the 50-percent jointing dates between the ITS-, the state-, and the CRD-level USDA/SRS averages. The biggest discrepancies between the ITS and ACC data are attributed to the difference in interpretation rather than to the location of the ITS within the CRD.

5.5.2 TEXAS (WINTER WHEAT)

Segment 1979, Deaf Smith County

Deaf Smith County is located in the west-central part of this CRD, which is in the Texas Panhandle. The minimum and maximum temperatures of record most representative of that area were obtained from Amarillo, Texas, approximately 64 kilometers (40 miles) east of the ITS and at a slightly lower elevation. The difference (warmer at the meteorological station because of the lower elevation) between the ITS temperature and the average temperature for the CRD would probably account for the slightly advanced CCEA crop calendar readings (plot 4, fig. 5-6).

Segments 1980 and 1978, Oldham and Randall Counties

These two ITS's are in close proximity to the nearest meteorological reporting station. Consequently, the minimum and maximum temperatures used to effect the adjustments will keep the ACC output in closer agreement with the ground truth (fig. 5-6.)

5.5.3 MINNESOTA (SPRING WHEAT)

Segment 1987, Polk County

The ACC was not run for Minnesota until June 24, 1975; consequently, no comparison was made through the jointing stage. Segment 1987, Polk County, is close to the center and should be representative of the CRD. The only discrepancy appears around the heading stage (figure 5-7). The meteorological data prior to the crop calendar adjustment date indicated unseasonably cool weather [with a -6° C (-21° F) deviation from the weekly normal temperature]. The NOAA *Weekly Weather and Crop Bulletin* for Minnesota covering the period of July 7 through 13, 1975, reported there was "small grain ripening in the southern two-thirds, but in important northern counties a lot of acreage not yet headed."

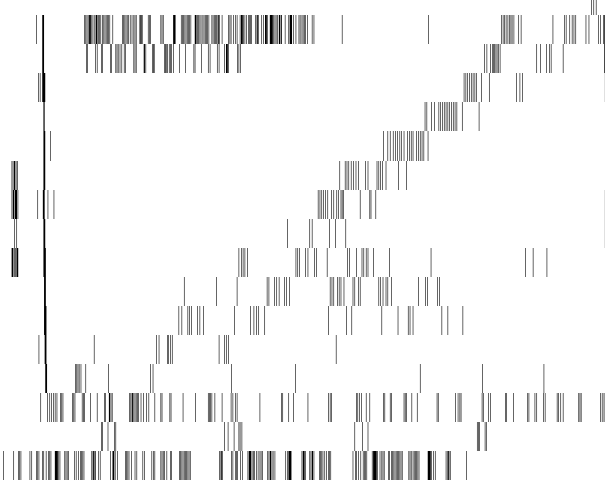
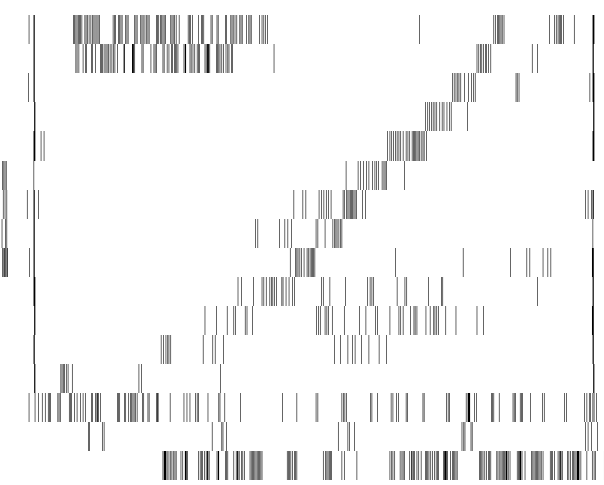
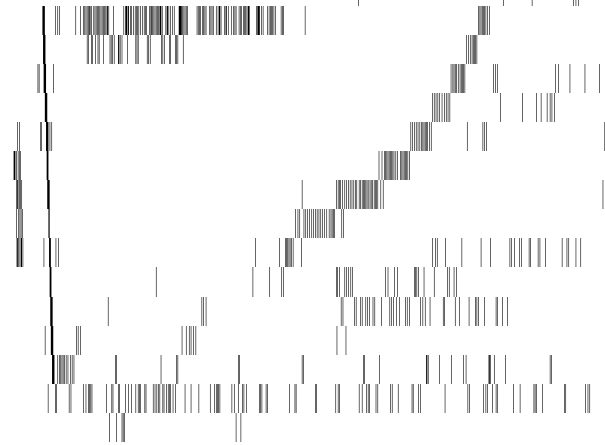
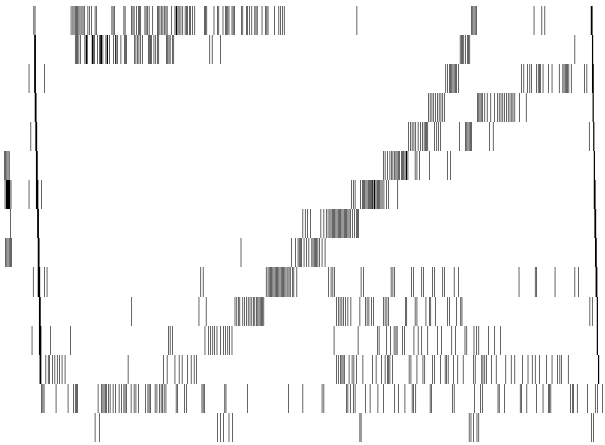
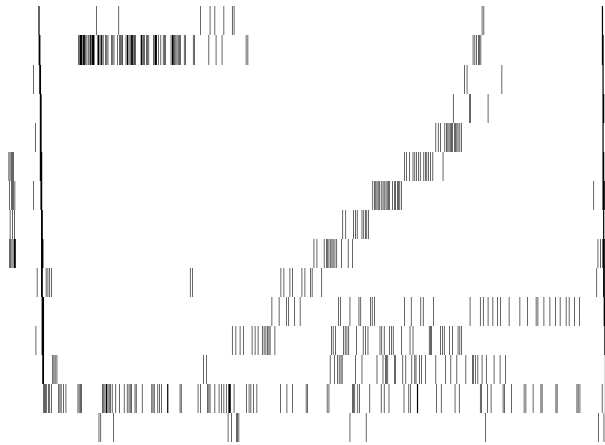
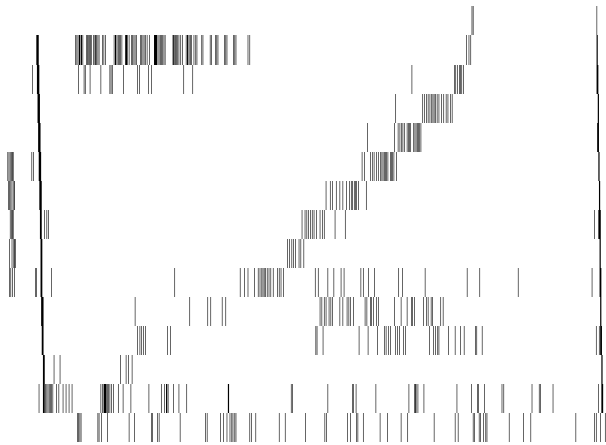
5.5.4 MONTANA (SPRING WHEAT)

Segment 1971, Hill County

The major difference between the ITS ground-truth data and the ACC output was the reported planting data for the CRD and for the ITS (fig. 5-7). The ACC model performed very well in the ITS throughout the season. This was a late season for Montana, which the ACC tracked very well.

Segments 1970 and 1969, Liberty and Toole Counties

Both of these ITS's are located in the northwest part and may not be representative of the other wheat-growing areas within the CRD. The most obvious discrepancy between the ground-truth data and



ACC plots is the fact that the Liberty County ground-truth crop calendar is consistently slower than the ACC (fig. 5-7). The Toole County plot (plot 4) is first fast and then slow after the heading stage. This suggests unusually large differences in the development of wheat between the two ITS's, which are located only approximately 48 kilometers (30 miles) apart. The fact that one is slower and the other faster than the ACC indicates that the ACC may indeed be providing a good average for that CRD. A comparison against the USDA/SRS CRD average confirms this. (The USDA/SRS CRD average is plotted on the Liberty County plot. It is noteworthy that the 50-percent dates for emergence and jointing were not made available and are not plotted.)

5.5.5 NORTH DAKOTA (SPRING WHEAT)

Segment 1965, Burke County

The ITS planting date was May 24, 1975; the USDA/SRS planting date for the CRD as supplied to the CCEA for comparison to the model was May 30. After allowances were made for the difference in planting dates, no significant differences were apparent for the remainder of the crop calendar.

Segment 1966, Williams County

This ITS is located in the center of the county, which is in the southwest part of the CRD. The meteorological input is provided by Williston, North Dakota, minimum and maximum temperature reports. The reports from this station are more representative of the ITS than of the CRD because of the station's close proximity to the ITS. Elevation differences are minimal. The CRD planting date supplied by USDA/SRS to start the ACC was May 30, 1975; the ITS planting date was May 21 (fig. 5-7). This difference in dates accounts for the difference in the initial development stages between the ITS and the ACC plot.

5.5.6 RESULTS OF ACC ANALYSES

To summarize the evaluations in sections 5.5.1 through 5.5.5, the ACC performance for Phase I operations during the jointing-to-soft-dough stage for winter wheat and the planting-to-soft-dough stage for spring wheat in the U.S. Great Plains appeared to be quite good, assuming the validity of planting dates. The biggest discrepancies were early in the season – at jointing for winter wheat and at planting for spring wheat. An 8- to 10-day disagreement occurred between the dates the USDA/SRS reported for the CRD (which were used as starter dates for the ACC) and the ITS ground-truth data. The ITS ground truth and ACC output were closest to agreement at the heading and soft-dough stages. Indications are that more accurate starter dates would have allowed the ACC to perform more accurately throughout the spring and summer.

The results of the study show that

- a. Accurate starter models for spring wheat are vital to good overall performance of the ACC.
- b. Proper operation of the ACC for winter wheat before and through dormancy to provide an accurate estimate of jointing in spring is vital to the overall operation of the ACC for winter wheat.

6. PHASE II SPECIAL STUDIES

This section contains a description of several special studies performed in Phase II. All of the ITS investigations were considered to be special studies even if they were similar to the blind site studies reported in section 4.

6.1 ITS STUDY OF THE DEPENDENCE OF CAMS ERROR ON TRUE WHEAT PROPORTIONS

The ITS's were not aggregated by CAS but they were processed by CAMS as if they were regular sample segments; i.e., an estimate of the small grains proportion within the ITS was made using Phase II classification procedures. The analyst selecting the training data did not have access to the ground truth data.

Winter Wheat

In Phase II there were 32 acquisitions from 14 winter wheat ITS's located in Kansas, Washington, Idaho, Texas and Indiana. The CAMS errors for these acquisitions are plotted as a function of ground truth wheat* proportion in figure 6-1. The overall trend is similar to that observed in the blind site data (figure 4-3), i.e., there is a trend toward negative values of $\hat{X} - X$ as X increases. In fact, for $X > 10$ percent there is only one acquisition for which the CAMS result is not an underestimate relative to ground truth. Similar results were found for the blind site data (section 4.2.2.1). The data points in figure 6-1 do not constitute a random sample since in many cases two or three of them correspond to different acquisitions of the same segment. Therefore, a statistical analysis of these data was not performed.

*The CAMS wheat proportions were obtained by ratioing the CAMS small grains proportions.

Spring Wheat

In Phase II there were 16 acquisitions from 10 spring wheat ITS's. There were two from ITS's in North Dakota, two in Montana, and one in Minnesota. The other 11 acquisitions were from three ITS's in Canada.

Figure 6-2 shows a plot of the CAMS classification errors as a function of ground truth proportions. There is a tendency toward negative values of $\hat{X} - X$ as X increases, but it is less well developed than in the spring wheat blind site data (section 4.2.2.2). In particular, five out of the fifteen points for $X > 25$ percent correspond to positive values of $\hat{X} - X$. A statistical analysis was not performed on these data for the same reason given above for the winter wheat data.

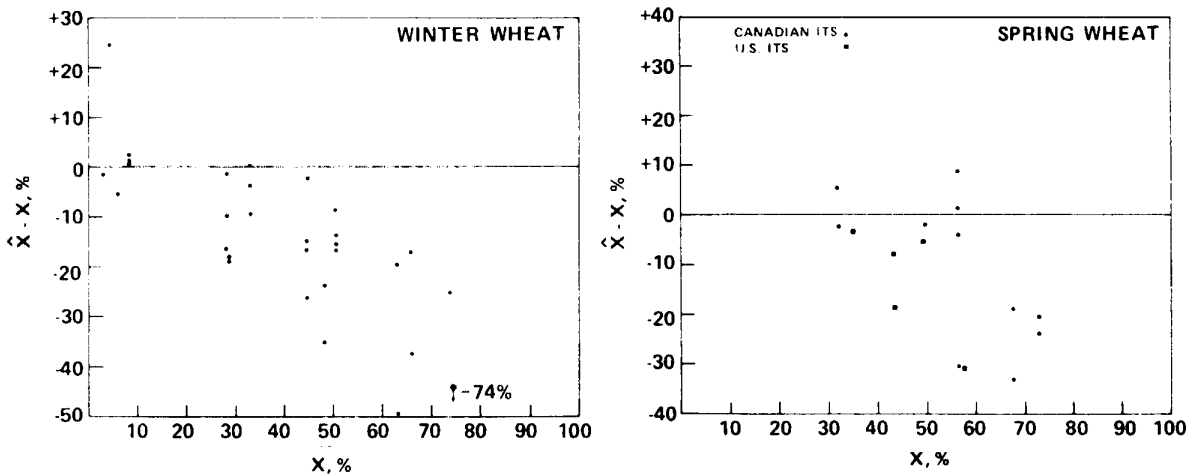


Figure 6-1.— Plot of CAMS classification error as a function of ground truth wheat proportions.

Figure 6-2.— Plot of CAMS classification error as a function of ground truth wheat proportions.

6.2 INVESTIGATION OF THE DEPENDENCE OF CAMS ERROR ON ACQUISITION DATE

In this section, "acquisition date" refers to the date of the last acquisition used to classify the CAMS data. The CAMS classifications were based on this acquisition and on all previous acquisitions. Two studies of the dependence of CAMS error on acquisition date were conducted in Phase II. One of these was an ITS investigation (section 6.2.1) and the other was a blind site investigation (section 6.2.2).

6.2.1 ITS INVESTIGATION

The data used in these investigations were the same as those used in the investigations reported in section 6.1 for both winter and spring wheat.

Winter Wheat

Figure 6-3 shows the plot of the winter wheat CAMS errors as a function of acquisition date. It will be seen that the estimates based on very early acquisitions (before December) have very large errors. For later acquisitions the only well developed trend seems to be a consistent underestimation. The overall average of $\hat{X} - X$ was -14.4 percent. When estimates based on acquisitions before December 1975 were omitted, the average of $\hat{X} - X$ was -9.6 percent.

Spring Wheat

Figure 6-4 shows the plot of the CAMS error as a function of the acquisition date for spring wheat. There is a clear tendency toward underestimation for early acquisitions and overestimation for late acquisitions. All the acquisitions before the first week in August led to underestimates and all the acquisitions after the first week in August led to overestimates.

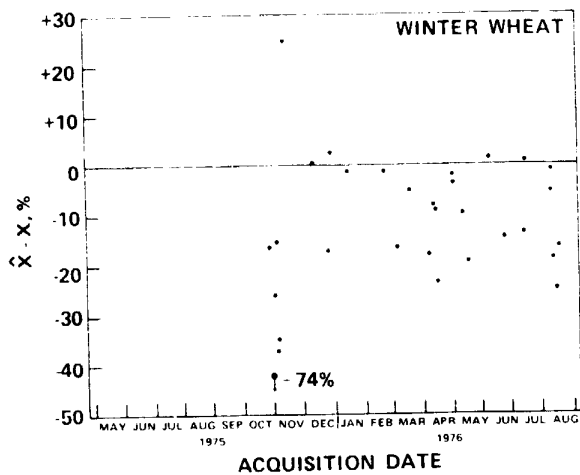


Figure 6-3.— Plot of CAMS error as a function of acquisition date for winter wheat.

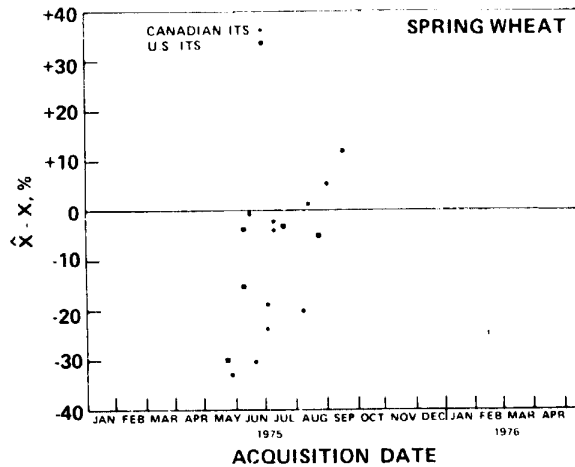


Figure 6-4.— Plot of CAMS error as a function of acquisition date for spring wheat.

6.2.2 BLIND SITE INVESTIGATION

In this investigation the average errors for blind site wheat proportions in the USGP were studied as a function of the month of the latest acquisition used by CAMS to obtain their estimate of wheat proportions. All of the winter wheat blind sites in the USGP for which data were available were used. Spring wheat was not studied because data were not available for enough segments.

Table 6-1 gives the mean squared error, the bias, and the standard deviation for each month from November 1976 to July 1977. Also given is the number of sites for each month. Each site used had at least one acquisition in that month. Since the same set of sites was not used for each month, some of the variation from month to month was due to a corresponding change in the sample. The most interesting result shown in table 6-1 is the large drop in the mean squared error and standard deviation in April, followed by an increase in May and June. The same trend was observed for most of

TABLE 6-1.-- FULL-MONTH CLASSIFICATION ERROR FOR WINTER WHEAT

Acquisition Period	MSE	Bias	Std Dev	Number of Sites
11/1 - 11/30	120.1	-4.5	10.1	36
12/1 - 12/31	161.8	-5.0	11.8	47
1/1 - 1/31	114.9	-5.5	9.3	61
2/1 - 2/29	123.5	-5.7	9.6	60
3/1 - 3/31	80.5	-1.3	8.9	64
4/1 - 4/30	45.2	-3.3	5.9	63
5/1 - 5/31	70.2	-0.9	8.4	82
6/1 - 6/30	84.3	-2.9	8.8	88
7/1 - 7/31	48.3	-0.6	7.0	58

TABLE 6-2.-- MID-MONTH TO MID-MONTH CLASSIFICATION ERROR FOR WINTER WHEAT

Acquisition Period	MSE	Bias	Std Dev	Number of Sites
11/16 - 12/15	85.1	-3.4	8.7	27
12/16 - 1/15	191.8	-7.0	12.1	42
1/16 - 2/15	110.0	-5.1	9.2	65
2/16 - 3/15	108.6	-4.2	9.6	73
3/16 - 4/15	57.7	-1.1	7.6	59
4/16 - 5/15	54.7	-1.3	7.3	80
5/16 - 6/15	72.9	-2.7	8.1	92
6/16 - 7/15	70.6	-2.1	8.2	66
7/16 - 8/15	36.5	0.0	6.1	31

the individual states. Also, there was a significant decrease in the magnitude of the bias in March.

Table 6-2 gives similar results with the exception that the acquisition windows were shifted by 15 days in an attempt to assess the effect of sampling. The same overall pattern exists except that in this case "minimum" in the mean squared error and standard deviation is spread over the period of March 16 through May 15 and the decrease in the bias is in the period of March 16 through April 15.

6.3 ITS STUDY OF LABELING AND CLASSIFICATION ERRORS

After the normal processing was completed for a given ITS, accuracy assessment personnel randomly selected approximately 15 wheat and 15 nonwheat test fields in the ground truthed area of the ITS. The ground truthed area was usually 3×3 miles and in any case was always smaller than the segment area (5×6 nautical miles). The test fields were selected so as not to overlap any of the training fields chosen by the analyst.

The test fields were used to determine the probability of correct classification (PCC) by comparing the classification results for these fields with ground truth on a pixel-by-pixel basis.

Labeling error was studied by determining the percentage of training fields in the ground truthed area that were labeled correctly. Usually there were only eight to ten such fields since, in general, less than one-half of the total number of training fields were in the ground truthed area.

Winter Wheat

Table 6-3 shows the results obtained in the final classification for the winter wheat ITS's.

Labeling accuracy was determined for seven ITS's. For non-small grains (NSG) the labeling accuracy was 100 percent for five of the six cases, but for small grains (SG) the labeling accuracy was 100 percent for only three of the six cases. In three cases the labeling accuracy for SG was less than that for NSG, and in one case the labeling accuracy for SG was greater than that for NSG. Thus, the labeling accuracy was considerably better for NSG than for SG.

The probability of correct classification was determined for 11 of the winter wheat ITS's. In all but one of these the PCC for NSG was higher than for SG, and the average value for SG (63 percent) was considerably lower than that for NSG (86.9 percent). Thus, the error of omission (classifying SG as NSG) is considerably larger than the error of commission (classifying NSG as SG).

The fact that the PCC for SG is 27 percent lower than that for NSG whereas the labeling accuracy for SG is only 10 percent below that for NSG suggests that the low value for the PCC for SG was probably due in part to the analysts missing some SG signatures. This is probably a major cause of the observed under-estimation.

Spring Wheat

Table 6-4 shows the results obtained in the final classification for the spring wheat ITS's in the U.S. and Canada. Training field labeling accuracy was not available for these sites.

TABLE 6-3.— ITS WINTER WHEAT FINAL CLASSIFICATION RESULTS

Segment	State	Acq	\hat{X}	X	X - X	PCC		Labeling Accuracy	
						SG	NSG	SG	NSG
1961	Kansas	2006	8.8	8.2	0.6	HC	HC	HC	HC
1962	Kansas	3645	49.0	66.1	-17.1	62.7	78.3	100	100
1963	Kansas	2346	34.0	50.7	-16.7	66.5	94.8	75	100
1964	Kansas	1276	42.7	44.9	-2.2	93.4	79.5	100	100
1988	Kansas	1276	29.2	33.0	-3.8	67.4	97.3	—	—
1972	Washington	2316	48.8	74.0	-25.2	53.2	100	—	—
1973	Washington	1786	29.9	44.7	-14.8	78.9	99.5	100	100
1974	Washington	1426	43.6	63.1	-19.5	42.5	58.7	—	—
1976	Idaho	2266	26.8	28.2	-1.4	52.3	53.7	75	67
1977	Idaho	2276	9.6	28.7	-19.1	47.9	99.3	75	100
1978	Texas	1106	24.7	48.4	-23.7	51.1	99.5	80	100
1980	Texas	0566	1.6	3.0	-1.4	HC	HC	HC	HC
1982	Indiana	2266	0.6	6.0	-5.4	HC	HC	HC	HC
1983	Indiana	3215	29.1	4.5	24.6	78.0	95.8	—	—
Average			27.0	35.9	-8.9	63.0	86.9	86	95

Acq = Acquisition date; last digit indicates year; e.g., 2006 indicates that the segment processed was the 200th day of 1976.

HC = indicates that a hand count was performed.

\hat{X} = CAMS small grains proportion estimate for the ground truthed area.

X = Ground observed proportion of small grains.

PCC = Estimate of the probability of correct classification.

SG = Small grains.

NSG = Non-small grains.

Labeling Accuracy = Percentage of training fields (in ground truthed area) correctly labeled.

TABLE 6-4.- ITS SPRING WHEAT FINAL CLASSIFICATION RESULTS

Segment	State/ County	Acq.	\hat{X}	X	$\hat{X} - X$	PCC	
						SG	NSG
1965	N. Dakota	2216	39.6	47.0	-7.4	48.6	97.9
1967	N. Dakota	1866	30.0	34.5	-4.5	—	—
1969	Montana	1566	28.0	45.0	-17.0	71.6	88.8
1971	Montana	1556	44.2	50.2	-6.0	94.8	95.4
1987	Minnesota	1456	45.8	56.2	-10.4	83.0	95.8
1958	Canada	2246	58.1	56.9	+1.2	92.8	89.0
1984	Canada	2436	38.2	33.2	+5.0	88.7	97.9
1985	Canada	1536	47.2	31.5	+15.7	95.8	92.9
1991	Canada	2186	53.0	72.9	-19.9	75.4	84.0
1995	Canada	1826	49.2	67.7	-18.5	86.9	99.2
Average			43.3	49.4	-6.1	81.9	93.4

Acq. = Acquisition date; last digit indicates year; e.g.,
2006 indicates that the segment processed was the
200th day of 1976.

\hat{X} = CAMS proportion estimate of small grains.

X = Ground observed proportion of small grains.

PCC = Estimate of the probability of correct classification.

SG = Small grains.

NSG = Non-small grains.

The probability of correct classification was determined for nine sites. In all but two of them the PCC for NSG was larger than for SG. The average for SG (81.9 percent) was smaller than the average for NSG (93.4 percent) but the difference was less than that obtained for winter wheat. Also, the spring wheat accuracies for both SG and NSG are considerably higher than the corresponding accuracies for winter wheat.

6.4 EFFECT OF BIOPHASE ON PROPORTION ESTIMATION

Two studies were conducted in Phase II to investigate the effect of biophase on proportion estimation. In one of these the bias and standard deviation of the proportion errors were estimated for blind sites analyzed using various biophase combinations. It is described in section 6.4.1. In the second study the Wilcoxon matched-pairs signed-rank test was used to investigate whether proportion estimation errors using data from biophase 4 were different from those using data from biophase 1.

6.4.1 EFFECT OF VARIOUS BIOPHASE COMBINATIONS

Table 6-5 shows estimates of the bias and standard deviation for various combinations of biophase. All the winter wheat blind sites in the USGP were used. Spring wheat blind sites were not studied because sufficient data were not available.

TABLE 6-5.- CLASSIFICATION ERROR BY BIOWINDOW COMBINATION
(WINTER WHEAT)

Combination	Bias	Std dev.	Number of Sites
1	-2.5	9.2	117
1-2	-0.8	6.8	72
1-3	-5.1	6.6	19
1-2-3	0.8	4.9	32
1-4	-6.1	14.1	19
1-2-4	-2.0	7.9	33
1-3-4	-5.5	6.6	17
1-2-3-4	+1.1	5.1	31

The best results were obtained using data from the biophase combinations 1-2 and 1-2-3. It will be seen that the last four combinations in table 6-5 are the same as the first four combinations except that biophase 4 has been added. In every case the magnitude of the bias and the standard deviation were increased by adding biophase 4 data, except for the combination 1-3, where the magnitude of the bias increased but the standard deviation remained the same. These results indicate that better estimates might be obtained if data from biophase 4 were not used.

6.4.2 BIOPHASE 1 VERSUS BIOPHASE 4

A test was made to determine whether the proportion estimates based on data from biophase 4 were significantly different from proportion estimates based on data from biophase 1. Since there were not enough paired data per state for biophases 1 and 4 for reliable comparison, the data for the five USSGP states were merged (i.e., for 23 blind sites) and a comparison of biophase data was made on this basis.

The Wilcoxon matched-pairs signed-rank test¹ was applied to \hat{X}_1 and \hat{X}_4 where \hat{X}_1 is the proportion of small grains estimated in a given blind site using biophase 1 data and \hat{X}_4 is a corresponding estimate using biophase 4 data.

The signed-rank test as applied here assumes that the differences $\hat{X}_1 - \hat{X}_4$ can be ordered in terms of a greater than or less than relation. Each rank is assigned the same algebraic sign as the

¹R.P. Runyon and A. Haber, Fundamentals of Behavioral Statistics, Addison-Wesley Publishing Co., Reading, Mass., 1971, pp. 263-265.

corresponding difference so that the direction as well as the magnitude of $\hat{X}_1 - \hat{X}_4$ is utilized in the test. The null hypothesis is made that the sums, T, of positive and negative ranks are equal with an assigned level of significance; i.e., positive and negative ranks of the same magnitude are equally likely.

Critical values of T are to be found in tables prepared by Wilcoxon¹ for various numbers, N, of samples (here N = 23). Under the null hypothesis the distribution of the differences $\hat{X}_1 - \hat{X}_4$ is symmetric about zero; i.e., a mistake of a given magnitude is equally likely using biophase 1 or 4.

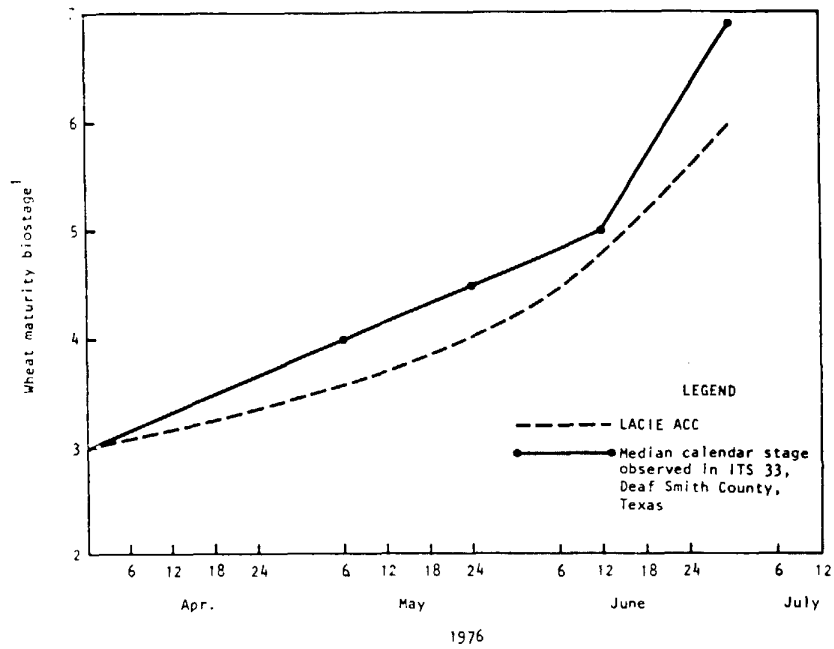
Upon applying the test described, for a 10-percent level of significance, it was found that the null hypothesis could not be rejected. It follows that LACIE estimates made using data from biophase 4 could not be said to be different from estimates made on the basis of data from biophase 1.

6.5 ADJUSTABLE CROP CALENDAR ERROR

The adjustable crop calendar is designed to indicate to the CAMS analyst the growth stage of wheat and other crops in the segments he is analyzing. It can therefore be expected to have a considerable impact on the accuracy of the CAMS estimates. A study was performed to determine the accuracy of the ACC by comparing it with ground-observed growth-stage data.

Ground-observed growth-stage data were collected by USDA/ASCS personnel over eight ITS's in Texas and Kansas during the months of April through June. These ground-observed data were plotted along with comparable LACIE ACC-predicted wheat development data. One of the plots (from Deaf Smith County, Texas) is presented in figure 6-5.

¹ Ibid, table J, p. 308.



¹According to the Robertson Biometeorological Time Scale, the numbered biostages are: 1 = planting, 2 = emergence, 3 = jointing, 4 = heading, 5 = soft dough, 6 = ripening, and 7 = harvest.

Figure 6-5.— Plot of observed and predicted progression of crop calendar stages for the Deaf Smith County, Texas ITS.

Table 6-6 shows the differences \bar{D} between the LACIE ACC estimates and the ground truth values for the sixth day of April, May, and June. A negative sign indicates the LACIE estimate was lower (i.e., "behind") the ground truth. It will be seen that in most cases the LACIE estimate was behind ground truth and that the difference got larger as the season progressed. In June all the ACC predictions were behind the ground truth stages.

TABLE 6-6.— COMPARISON OF LACIE ADJUSTABLE CROP CALENDAR WITH OBSERVED STAGES IN THE EIGHT INTENSIVE TEST SITES IN THE U.S. SOUTHERN GREAT PLAINS

(\bar{D} in the BMTS units of the Robertson scale)

Site		Date		
County	State	April 6	May 6	June 6
Randall	Texas	-0.12	-0.33	-0.28
Deaf Smith	Texas	-.08	-.42	-.39
Oldham	Texas	.01	0	-.08
Ellis	Kansas	0	-.42	-.51
Rice	Kansas	0	-.44	-.38
Phinney	Kansas	-.17	-.04	-.38
Saline	Kansas	-.18	-.51	-.42
Morton	Kansas	-.16	0	-.08
Average		-.12	-.27	-.32

6.6 RELATION OF CAMS ERROR TO CROP CALENDAR ERROR

This investigation was performed to determine whether crop calendar error had an influence on the accuracy of CAMS estimates.

All of the ITS acquisitions described in section 6.1 which had crop calendar data were used. The classification errors were regressed on the crop calendar errors (measured in days). The correlation coefficients are shown in table 6-7. Significance tests applied to the correlation coefficients indicated that no significant correlation existed between crop calendar error and classification error for any of the four cases shown in table 6-7.

TABLE 6-7.— CORRELATION OF CROP CALENDAR ERRORS AND CLASSIFICATION ERRORS

	Winter wheat		Spring wheat	
	Sample size	r	Sample size	r
Adjustable crop calendar	9	.57	12	-.37
Nominal crop calendar	10	.27	13	.10

6.7 SUMMARY OF PHASE II TEST AND EVALUATION OF YIELD MODELS*

Eleven years of test yield predictions for the LACIE Great Plains model zones were evaluated for their combined and individual performances. The estimates were generated with the CCEA regression models as revised for LACIE Phase II with a "flagging" procedure for weather inputs and new trend segments. Also, characteristics of individual models were analyzed to identify first-order sources of strengths and weaknesses.

The hypothesis of the 11 years of simulated yield predictions meeting the LACIE 90/90 criterion was tested with a sign test. The hypothesis was accepted for the criterion applied at the country level, but was rejected with application of the criterion directly to the Great Plains area. Projection of the 90/90 criterion to individual zones may not be valid since yield errors for several zones appeared positively correlated.

*Details of these tests are reported in the LACIE document: Phase II Test and Evaluation of Yield Models for the U.S. Great Plains.

Three of the models showed a significant mean level bias which was attributed to differences between areas used to develop and test the models.

A check was made using the Phase II (1976) case to reconfirm that there are no apparent differences between applying the models at the district level or applying them to weather aggregated to the state level.

All but two of the models displayed a significant tendency to overestimate when yields were low and vice versa (a type of functional bias seen as restricted dynamic ranges).

Estimates by the complete weather versions of the Red River, Montana winter wheat and Colorado models did not produce mean square errors significantly smaller than the trend-only versions. Then, in a comparison using constant trend coefficients, the mean square errors for all zones were smaller than when the coefficients were recomputed after each additional year entered the regression. The coefficients for trend terms appeared to be the least stable.

APPENDIX A

PHASE II ACCURACY ASSESSMENT METHODOLOGY

APPENDIX A

PHASE II ACCURACY ASSESSMENT METHODOLOGY

A.1 INTRODUCTION

This appendix contains mathematical details of the techniques used in accuracy assessment. The methods used in comparing the LACIE estimates for acreage, yield, and production with the reference standard are presented in section A.2. The techniques used to study errors in the LACIE estimates are discussed in section A.3.

A.2 COMPARISON OF LACIE ESTIMATES WITH REFERENCE STANDARDS

The reference standards to which the LACIE estimates are compared are the USDA/SRS estimates for the United States and the FAS estimates for foreign countries. The statistic used for making these comparisons is the relative difference (RD) defined as follows:

$$RD = \left(\frac{LACIE - STANDARD}{LACIE} \times 100\% \right)$$

where LACIE stands for the LACIE estimate of wheat production, area, or yield and STANDARD represents the corresponding reference standard estimate. This definition expresses the difference between the two estimates as a percentage of the LACIE estimate.

Significance tests of no difference are made only at the region or country level for the LACIE production, area, and yield estimates for spring wheat, winter wheat, and total wheat. For a significance test, the LACIE estimate (of wheat production, area, or yield) is assumed to be approximately normally distributed with unknown mean μ and variance σ_{LACIE}^2 . A test of the hypothesis

$$H_0 : \mu = STANDARD$$

versus the alternative hypothesis

$$H_A : \mu \neq STANDARD$$

is then made using this assumption. The test statistic is given by

$$z = \frac{\text{LACIE} - \text{STANDARD}}{\hat{\sigma}_{\text{LACIE}}} \quad (\text{A-1})$$

which, under the null hypothesis, is approximately normally distributed with mean 0 and variance 1. The null hypothesis is rejected in favor of the alternative at the α -level of significance if

$$|z| > z_{\alpha/2}$$

where $z_{\alpha/2}$ is the $(1 - \frac{\alpha}{2})$ critical point of the standard normal distribution. For $\alpha = 0.10$, $z_{\alpha/2} = 1.645$, and if $|z| > 1.645$, it is concluded that the mean of the LACIE estimator is significantly different from the reference standard estimate.

A.3 ERROR SOURCES IN LACIE

The techniques used to study errors in the estimates of acreage, yield, and production are discussed respectively in section A.3.1, A.3.2, and A.3.3 of this appendix.

A.3.1 ACREAGE

This section contains a description of the methods used to estimate the following:

1. The errors in segment wheat proportion estimates (section A.3.1.1).
2. Wheat acreage at the state and higher levels (section A.3.1.2).
3. The variance of the wheat acreage estimates (section A.3.1.3).
4. The bias in the acreage estimates for large areas having ground truth available for a subset of their LACIE segments (section A.3.1.4).

5. The relative variances of the sampling and classification errors in stratum wheat acreage estimates (section A.3.1.5).

A.3.1.1 Error in Proportion Estimates at the Segment Level

This section describes the statistical calculations used to compare CAMS wheat proportion estimates for blind sites with the corresponding ground truth values. Let N be the number of segments allocated to a region (state or higher level) and let n be the number of blind sites selected randomly from these N segments. For a region, let \hat{X}_i represent the CAMS estimate of the proportion of wheat in the i th segment and let X_i represent the ground truth proportion of wheat in the i th segment, where $i = 1, \dots, N$. Then the average error μ_D is given by

$$\mu_D = \frac{1}{N} \sum_{i=1}^N (\hat{X}_i - X_i) \quad (\text{A-2})$$

The estimate of μ_D is given by

$$\bar{D} = \frac{1}{n} \sum_{i=1}^n (\hat{X}_i - X_i) \quad (\text{A-3})$$

where the summation is taken over the n blind sites. Letting $D_i = \hat{X}_i - X_i$, we may estimate the variance of \bar{D} by

$$s_{\bar{D}}^2 = \left(\frac{1}{n} - \frac{1}{N} \right) \frac{\sum_{i=1}^n (D_i - \bar{D})^2}{n-1} \quad (\text{A-4})$$

Lower and upper confidence limits for the population average difference μ_D are given by

$$\mu_{D_L} = \bar{D} - t_{1-\alpha/2} s_{\bar{D}}, \quad \mu_{D_U} = \bar{D} + t_{1-\alpha/2} s_{\bar{D}} \quad (\text{A-5})$$

where $t_{1-\alpha/2}$ is the value of the $1-\alpha/2$ percentage point, from the Student's t distribution with $(n-1)$ degrees of freedom, corresponding to the desired confidence level of $1-\alpha$.

The hypothesis $\mu_D = 0$ (i.e., no bias) is rejected at the α -level of significance if $\left| \frac{\bar{D}/S_{\bar{D}}}{\bar{D}} \right| > t_{1-\alpha/2}$, or equivalently, if the confidence interval given by equation (A-5) does not contain zero.

A.3.1.2 Acreage Estimation

This section gives a brief summary of the methods used to estimate wheat acreage. These methods are described in detail in appendix B of the CAS Requirements Document.*

A.3.1.2.1 Background of Sample Allocation

The LACIE sample allocation in the U.S. Great Plains (USGP) region is based upon a two-stage stratified sampling scheme in which counties represent the primary sampling units (substrata) and 5- x 6-nautical-mile segments are secondary sampling units. The criterion for determining the total sample size was the ability to achieve a sampling error of 2 percent or less for the country wheat acreage estimates and, hopefully, the ability to meet the 90/90 criterion goal for the production estimate.

Sample segments were allocated to the counties based on relative weights derived from agriculture and wheat acreage reported in 1969 agriculture census statistics. Depending upon the relative weights, counties were designated as Group I (at least one sample segment in the county), Group II (at most one sample segment in a county), or Group III (no sample segments in the county). All Group II counties in a CRD (stratum) were combined to determine the number of segments allocated to the Group II part of the CRD.

*Crop Assessment Subsystem (CAS) Requirements Vol IV (Rev. B) (Change Notice, March 8, 1977), JSC-11329, LACIE C00200.

In this appendix any reference to the CAS Requirements Document indicates this specific document.

A probability proportional to size (PPS) procedure was applied to select the Group II counties in a CRD which were to receive these segments.

Once the number of segments to be allocated to each county was determined, the sample segments were selected at random within the agricultural area of the county. For further details of the LACIE sampling scheme refer to the CAS Requirements Document (JSC-11329).

A.3.1.2.2 Aggregation of Acreage Estimates

Wheat acreage estimates are made for each CRD, state, and region (group of states) in the USGP. However, no estimate is made for a state if it does not contain three or more segments satisfactorily processed by CAMS. Segment data may be lost due to the following cases of nonresponse:

1. The sample segment being obscured by cloud cover.
2. Landsat data quality being insufficient to permit processing.
3. Landsat data acquisition failing to register with the reference Landsat image.
4. Failure of acquisition/processing procedures to provide an acceptable estimate.

No replacement is allowed if a sample segment is not workable by CAMS.

A CRD acreage estimate consists of three components:

1. An acreage estimate for the Group I counties in the CRD for which segment data exist. (A group I county is treated as a Group III county if it does not have at least one segment with an acceptable proportion estimate.)
2. An acreage estimate for the entire set of Group II counties in the CRD if there is at least one segment with an acceptable

proportion estimate in this set of counties. (Otherwise, the Group II counties are all treated as Group III counties.)

3. An acreage estimate for the Group III counties, including the Group I and Group II counties being treated as Group III counties.

The wheat acreage estimates for these three components are computed using a stratified random sampling estimator for the Group I counties, a PPS estimator for the Group II counties, and a ratio estimator for the Group III counties.*

There are three categories of Group III acreage estimates, depending on the number of segments in a CRD for which data are available. Categories 1, 2, and 3 correspond respectively to three or more segments, one or two segments, and no segments having data available. The ratio used for the Group III estimator is the ratio of historical wheat acreages for Group III counties to Group I and Group II counties. For category 1 estimates it is based on acreages in the CRD. For category 2 and category 3 estimates it is based on acreages in the state containing the CRD for which the estimate is being made.

The CRD wheat acreage estimate is obtained from the sum of the wheat acreage estimates for Group I, II, and III counties. Next, aggregation of the CRD acreage estimates gives a state wheat acreage estimate, and summation of the state acreage estimates gives the regional wheat acreage estimate. For specific aggregation formulas, see appendix B in the Cas Requirements Document.

In a mixed wheat area, separate aggregations are performed for spring and winter wheat and the total wheat acreage estimate is obtained by summing the results. This is done at the CRD and higher levels.

*For details on these standard estimation procedures, see Sampling Techniques by W.G. Cochran, Wiley, 1963.

A.3.1.3 Acreage Variance Estimation

The acreage variance estimation for a CRD requires an estimate of within-county variance for each of the Group I and Group II counties in the CRD. Often there is only one sample segment in a county and hence no direct estimate of the within-county variance is possible. Therefore, an indirect method is employed. This method uses a regression approach and is based on the assumption that the historical county proportions are well correlated with the CAMS proportions. The method consists of (1) forming homogeneous groups of counties in a state with respect to the within-county variability, (2) performing regression for the CAMS segment wheat proportion estimate onto the county historical wheat proportion, and (3) taking the residual mean square error (MSE) for an estimate of the within-county variance for each county in the group. This procedure for LACIE Phase II is described in appendix B of the CAS Requirements Document.

For estimation of a CRD acreage variance, the acreage variance components for Group I and Group II counties are estimated independently. For Group I counties it is computed according to the variance formula for a stratified random sampling scheme.¹ The appropriate inputs of county sizes, number of sample segments, and within-county variance estimates are obtained using the above-mentioned procedure. Similarly, the variance formula for a PPS estimator¹ is employed to compute the Group II acreage variance estimate. It requires all of the inputs mentioned in the Group I case plus the probabilities of selection of Group II counties for sample allocation. These probabilities are those utilized in determining which of the Group II counties in a CRD receive sample segments.

¹Cf = Sampling Techniques, by W. G. Cochran, Wiley, 1963.

The acreage variance component for the Group III counties depends directly on Groups I and II variances and contributes to the CRD acreage variance indirectly through the ratio utilized to obtain the Group III acreage estimate. The formulas used to calculate the acreage variance for the Group III counties are described in appendix B of the CAS Requirements Document. As mentioned above, there are three categories of Group III acreage estimates and each category has a different formula for the variance estimate. For category 1 the variance estimate depends on the acreage estimates for all the Group I and Group II counties in the CRD; for categories 2 and 3 it depends on the acreage estimates for all of the Group I and Group II counties in the state.

If data are available for at least three segments in each CRD in the state, the acreage variance estimate is computed by adding the variance estimates for the CRD's in the state. Otherwise, the state variance estimate is obtained using an aggregation procedure which accounts for the dependence between various CRD acreage estimates in a state.

Since the state acreage estimates are obtained independently, the acreage variance estimates at both the regional and country levels are computed by adding the state acreage variance estimates.

In a mixed wheat area, separate aggregations are performed for estimating the variance of the spring and winter wheat acreage estimates at the CRD and higher levels. In each case the estimation procedure is the same as that described above for each aggregation level. The acreage variance estimates at the CRD and state levels for the total wheat case are obtained from the previously described variance formulas using total wheat acreage estimates for sample segments and the historical total wheat for

counties in the area. For higher levels the total wheat acreage variance estimates are computed by taking the sum of the variance estimates for the states involved. The CRD and state level variance estimates for the total wheat case are not unbiased; therefore, the method of determining variance of a total wheat acreage estimate in a mixed wheat area is considered approximate.

A.3.1.4 Acreage Bias Estimation

The method for estimating bias described in this section is valid for any area having a sufficient number of blind sites to represent the bias. In this report it is applied at the state and higher levels.

The LACIE estimate of wheat acreage for a given area can be written

$$\hat{A} = \sum_{i=1}^n W_i \hat{X}_i \quad (\text{A-6})$$

where \hat{A} is the estimated wheat acreage, \hat{X}_i is the wheat proportion estimate in the i th LACIE segment, n is the number of processed LACIE segments, and $\{W_i\}_{i=1}^n$ are weights based on historical and cartographic data.*

Corresponding to \hat{A} is the true acreage, A , which can be written

$$A = \sum_{i=1}^n W_i^* C_i \quad (\text{A-7})$$

*The precise definition of W_i depends on whether the i th segment is used as part of a Group III estimate.

where C_i is the true wheat acreage for the county containing the i th segment and W_i^* is the value of the weight which would give perfect Group III estimates of wheat acreage for unsampled counties.

We can now write

$$\begin{aligned}\hat{X}_i &= C_i + (X_i - C_i) + (\hat{X}_i - X_i) \\ &= C_i + \delta_i + \epsilon_i\end{aligned}$$

where X_i is the true wheat proportion of the i th segment, δ_i is the sampling error and ϵ_i is the classification error. Since sampling is unbiased, we assume $E(\delta_i) = 0$; however, we do not assume unbiased classification. Instead, let θ be an average segment bias; i.e.,

$$E(\epsilon_i) = \theta$$

The bias in A is defined by $E(\hat{A} - A)$, which is thus given by

$$\begin{aligned}B = E(\hat{A} - A) &= E\left(\sum_{i=1}^n W_i \hat{X}_i - \sum_{i=1}^n W_i^* C_i\right) \\ &= \sum_{i=1}^n W_i E(C_i + \delta_i + \epsilon_i) - \sum_{i=1}^n W_i^* C_i \\ &= \sum_{i=1}^n (W_i - W_i^*) C_i + \theta \sum_{i=1}^n W_i\end{aligned}\tag{A-8}$$

Note that the first term of equation (A-8) represents a bias caused by the failure of the Group III ratios to be exact;

(i.e., $W_i \neq W_i^*$), whereas the second term is the average segment bias multiplied by the sum of the W_i .

At present, only the second term of equation (A-8) will be estimated, since good county-level data are not available for estimating the first term. The second term is estimated by (1) breaking up the large area into strata (not necessarily connected) for which the bias is assumed to be approximately

constant; (2) estimating θ_k by $\hat{\theta}_k = \frac{1}{n_k} \sum_{i=1}^{n_k} (\hat{X}_i - X_i)$, the average proportion error on a segment level in the k th stratum; and (3) aggregating $\hat{\theta}_k$ over the strata.

If \hat{B} represents the AA estimate of bias due to classification, a 90-percent confidence interval for B , the real bias, can be constructed by

$$(\hat{B} - 1.645\sigma, \hat{B} + 1.645\sigma)$$

where σ^2 is an estimate of the variance of \hat{B} .

If we assume $\text{Var}(\varepsilon_i) = \sigma_{ck}^2$ (a constant) within the k th stratum, then σ_{ck}^2 can be estimated by

$$\hat{\sigma}_{ck}^2 = \sum_{i=1}^{n_k} \frac{(\hat{X}_i - X_i - \hat{\theta})^2}{n_k - 1}$$

and $\text{Var}(\hat{B})$ can be estimated by

$$\hat{\text{Var}}(\hat{B}) = \sum_k \hat{\sigma}_{ck}^2 \left(\sum_{i=1}^{n_k} W_{ki} \right)^2$$

where W_{ki} is the weight for the i th segment in the k th stratum.

A.3.1.5 Contribution of Sampling and Classification to Acreage Estimation Error

This section describes the calculation of the contribution of sampling and classification errors to the variance of the LACIE production estimate.

A.3.1.5.1 Approach

The variance of the LACIE acreage estimate for a large area (e.g., zone) can be written

$$V^2 = \sum_i V_i \sigma_i^2$$

where σ_i^2 is the variance of the acreage estimate for the *i*th county and V_i is a weight which depends on the size of the county, the number of segments in the county, etc. (Refer to CAS Requirements Document, appendix B for details.)

The variance σ_i^2 represents a mean-squared deviation between the LACIE estimate for the county wheat proportion and the true county wheat proportion. This variance is caused mainly by two factors: sampling errors and classification errors.

In accuracy assessment, it is desirable to quantify the contribution of each of these error sources to the large area production estimate. The LACIE production estimate depends on acreage and yield estimation errors in a complicated way; hence, it is unrealistic to assume the error in the production estimate can be written as a sum of uncorrelated random variables representing acreage and yield errors. Instead, the effect of a particular error source is measured by the reduction in the LACIE production variance which would be achieved if that source were eliminated.

It will be assumed (section A.3.1.5.2) that the i th county acreage error variance σ_i^2 can be written $\sigma_i^2 = \sigma_c^2 + \lambda^2 \sigma_s^2$, where σ_c^2 is a contribution due to classification, and $\lambda^2 \sigma_s^2$ is a contribution due to sampling. To determine the effect of no classification error, the variance of the LACIE production estimate will be calculated using $\rho \sigma_i^2$ instead of σ_i^2 where ρ is

an estimate of the ratio $\frac{\lambda^2 \sigma_s^2}{\sigma_c^2 + \lambda^2 \sigma_s^2}$. Similarly, the effect of no

sampling error is estimated by replacing σ_i^2 by $(1 - \rho) \sigma_i^2$. This procedure is described in detail in section A.3.3.5 of this appendix. The following two sections describe the methods employed for estimating sampling and classification variances and the function ρ .

A.3.1.5.2 Acreage Regression Models

For counties with one sample segment, the LACIE estimate of the i th county wheat proportion can be written

$$\begin{aligned} \hat{X}_i &= C_i + (X_i - C_i) + (\hat{X}_i - X_i) \\ &= C_i + \epsilon_i + \delta_i \end{aligned} \tag{A-9}$$

where

\hat{X}_i = LACIE estimate of the wheat proportion in the sampled segment

C_i = true (current year) proportion of wheat in the county

X_i = true proportion of wheat in the sampled segment

ϵ_i = sampling error = $X_i - C_i$

δ_i = classification error = $Y_i - X_i$

It will be assumed that for some reasonably large area (e.g., a zone) the errors ϵ_i and δ_i have the following properties:

ϵ_i and δ_i are uncorrelated

$$E(\epsilon_i) = 0$$

$$E(\delta_i | X_i) = \lambda * X_i + \theta$$

$$V(\epsilon_i) = \sigma_s^2$$

$$V(\delta_i | X_i) = \sigma_c^2$$

It is also assumed that there is a linear model relating the current year county proportions, C_i , to the historical proportions which will be denoted by Z_i ; i.e.,

$$C_i = \alpha + \beta Z_i + \zeta_i \quad (\text{A-10})$$

where $E(\zeta_i) = 0$, $V(\zeta_i) = \sigma_H^2$, $\text{Cov}(\zeta_i, \epsilon_i) = \text{Cov}(\zeta_i, \delta_i) = 0$, and α and β are regression coefficients.

From the above assumptions and definitions, three basic regression models are obtained:

- a. True segment proportion versus historical county proportion - from the definition of ϵ_i ,

$$\begin{aligned} X_i &= C_i + \epsilon_i \\ &= \alpha + \beta Z_i + \zeta_i + \epsilon_i \end{aligned} \quad (\text{A-11})$$

It follows that

$$E(X_i) = \alpha + \beta Z_i \quad (\text{A-12})$$

$$V(X_i) = \sigma_H^2 + \sigma_s^2 \quad (\text{A-13})$$

- b. LACIE segment proportion versus ground truth segment proportion - from the definition of δ_i

$$\hat{X}_i = X_i + \delta_i \quad (\text{A-14})$$

It follows that

$$E(\hat{X}_i | X_i) = X_i + \lambda^* X_i + \theta \quad (\text{A-15})$$

$$V(\hat{X}_i | X_i) = \sigma_C^2 \quad (\text{A-16})$$

Writing $\lambda = 1 + \lambda^*$, one obtains

$$E(\hat{X}_i | X_i) = \lambda X_i + \theta \quad (\text{A-17})$$

$$V(\hat{X}_i | X_i) = \sigma_C^2 \quad (\text{A-18})$$

- c. LACIE segment proportion versus historical county proportion - from equations (A-12) through (A-18),

$$E(\hat{X}_i) = E_{X_i} (E(\hat{X}_i | X_i)) = E_{X_i} (\lambda X_i + \theta) = \lambda(\alpha + \beta Z_i) + \theta \quad (\text{A-19})$$

$$V(\hat{X}_i) = E_{X_i} (V(\hat{X}_i | X_i)) + V_{X_i} (E(\hat{X}_i | X_i)) = \sigma_C^2 + \lambda^2 (\sigma_H^2 + \sigma_S^2) \quad (\text{A-20})$$

As stated previously, one would like to estimate $\rho = \frac{\lambda^2 \sigma_S^2}{\sigma_C^2 + \lambda^2 \sigma_S^2}$.

None of the three regression models permits an estimate of σ_S^2 separately from σ_H^2 ; i.e., one can only estimate $\sigma_S^2 + \sigma_H^2$, not σ_S^2 alone. If current year county proportions C_i were available, σ_H^2 could be estimated, but since this is not the case,

$\rho^* = \frac{\lambda^2 (\sigma_s^2 + \sigma_H^2)}{\sigma_c^2 + \lambda^2 (\sigma_s^2 + \sigma_H^2)}$ will be estimated instead of ρ . If

$\sigma_H^2 \ll \sigma_s^2$ (a reasonable assumption) then $\rho^* \approx \rho$.

A.3.1.5.3 Normality Assumptions – Maximum Likelihood Estimation of ρ^*

Suppose a given zone has m blind site segments and n ordinary (i.e., not blind site) segments, and let the blind site segments be numbered 1 to m . It is assumed that ground truth wheat proportions $\{X_i\}_{i=1}^m$ are available for the blind sites and LACIE estimates $\{\hat{X}_i\}_{i=1}^{m+n}$ are available for all the segments. It is also assumed that historical wheat proportions $\{Z_i\}_{i=1}^{m+n}$ are available for the counties containing the segments. If $\sigma_H^2 \ll \sigma_s^2$ so that $\rho \approx \rho^*$ the regression models equations (A-11 through A-20) can be used to obtain

$$E(X_i) = \alpha + \beta Z_i; V(X_i) = \sigma_s^2 \quad i = 1, \dots, m$$

$$E(\hat{X}_i | X_i) = \lambda X_i + \theta; V(\hat{X}_i | X_i) = \sigma_c^2 \quad i = 1, \dots, m$$

$$E(\hat{X}_i) = \theta + \lambda \alpha + \lambda \beta Z_i; V(\hat{X}_i) = \lambda^2 \sigma_s^2 + \sigma_c^2 \quad i = m+1, m+n$$

If there is one segment per county, then the errors ϵ_i and δ_i are independent for different values of i , and hence the likelihood function of the sample can be written

$$L = \prod_{i=1}^m f(X_i, \hat{X}_i) \prod_{i=m+1}^{m+n} h(\hat{X}_i) \quad (A-21)$$

where $f(X_i, \hat{X}_i)$ is the joint density of X_i and \hat{X}_i for $i = 1, \dots, m$ and $h(\hat{X}_i)$ is the density of \hat{X}_i for $i = m+1, \dots, m+n$.

The function $\prod_{i=1}^m f(X_i, \hat{X}_i)$ can be written $\prod_{i=1}^m f(X_i, \hat{X}_i) =$

$\prod_{i=1}^m f(\hat{X}_i | X_i) g(X_i)$ where $f(\hat{X}_i | X_i)$ is the conditional density of \hat{X}_i given X_i and $g(X_i)$ is the density function of X_i .

If normality is assumed, $\prod_{i=1}^m f(X_i, \hat{X}_i) = \prod_{i=1}^m \frac{1}{\sigma_c \sqrt{2\pi}}$

$$\exp\left\{-\frac{1}{2\sigma_c^2} \sum_{i=1}^m (\hat{X}_i - \lambda X_i - \theta)^2\right\} \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma_s^2} \sum_{i=1}^m (X_i - \alpha - \beta Z_i)^2\right\}$$

and

$$\prod_{i=m+1}^{m+n} h(\hat{X}_i) = \frac{1}{(\lambda^2 \sigma_s^2 + \sigma_c^2)^{1/2} \sqrt{2\pi}} \exp\left\{-\frac{1}{2(\lambda^2 \sigma_s^2 + \sigma_c^2)} \sum_{i=m+1}^{m+n} (\hat{X}_i - \lambda\alpha - \theta - \lambda\beta Z_i)^2\right\}$$

Letting $Q = -2\log L - \log 2\pi$,

$$Q = m \log \sigma_s^2 + m \log \sigma_c^2 + n \log(\sigma_c^2 + \lambda^2 \sigma_s^2) + \frac{D_m}{\sigma_c^2} + \frac{T_m}{\sigma_s^2} + \frac{T_n}{\sigma_c^2 + \lambda^2 \sigma_s^2}$$

(A-22)

where

$$D_m = \sum_{i=1}^m (\hat{X}_i - \lambda X_i - \theta)^2$$

$$T_m = \sum_{i=1}^m (X_i - \alpha - \beta Z_i)^2$$

$$T_n = \sum_{i=m+1}^{m+n} (\hat{X}_i - \lambda\alpha - \theta - \lambda\beta Z_i)^2$$

One attempts to maximize L by finding a stationary point of Q:

$$-\frac{1}{2} \frac{\partial Q}{\partial \alpha} = \frac{\sum_{i=1}^m (X_i - \alpha - \beta Z_i)}{\sigma_s^2} + \frac{\sum_{i=m+1}^{m+n} \lambda (\hat{X}_i - \lambda \alpha - \theta - \lambda \beta Z_i)}{\sigma_c^2 + \lambda^2 \sigma_s^2} = 0 \quad (A-23)$$

$$-\frac{1}{2} \frac{\partial Q}{\partial \beta} = \frac{\sum_{i=1}^m Z_i (X_i - \alpha - \beta Z_i)}{\sigma_s^2} + \frac{\sum_{i=m+1}^{m+n} \lambda Z_i (\hat{X}_i - \lambda \alpha - \theta - \lambda \beta Z_i)}{\sigma_c^2 + \lambda^2 \sigma_s^2} = 0 \quad (A-24)$$

$$-\frac{1}{2} \frac{\partial Q}{\partial \theta} = \frac{\sum_{i=1}^m (\hat{X}_i - \lambda X_i - \theta)}{\sigma_c^2} + \frac{\sum_{i=m+1}^{m+n} (\hat{X}_i - \lambda \alpha - \theta - \lambda \beta Z_i)}{\sigma_c^2 + \lambda^2 \sigma_s^2} = 0 \quad (A-25)$$

$$-\frac{1}{2} \frac{\partial Q}{\partial \lambda} = \frac{\sum_{i=1}^m X_i (\hat{X}_i - \lambda X_i - \theta)}{\sigma_c^2} + \frac{-n\lambda \sigma_s^2 + \sum_{i=m+1}^{m+n} (\beta Z_i + \alpha) (\hat{X}_i - \lambda \alpha - \theta - \lambda \beta Z_i)}{\sigma_c^2 + \lambda^2 \sigma_s^2} + \frac{\lambda^2 \sigma_s^2 T_n}{(\sigma_c^2 + \lambda^2 \sigma_s^2)^2} = 0 \quad (A-26)$$

$$\frac{\partial Q}{\partial \sigma_c^2} = \frac{m}{\sigma_c^2} + \frac{n}{\lambda^2 \sigma_s^2 + \sigma_c^2} - \frac{D_m}{\sigma_c^4} - \frac{T_n}{(\lambda^2 \sigma_s^2 + \sigma_c^2)^2} = 0 \quad (A-27)$$

$$\frac{\partial Q}{\partial \sigma_s^2} = \frac{m}{\sigma_s^2} + \frac{n\lambda^2}{\lambda^2 \sigma_s^2 + \sigma_c^2} - \frac{T_m}{\sigma_s^4} - \frac{T_n \lambda^2}{(\sigma_c^2 + \lambda^2 \sigma_s^2)^2} = 0 \quad (A-28)$$

Equations (A-23) through (A-29) must be solved for the parameters α , β , θ , λ , σ_c^2 , and σ_s^2 . If $\hat{\alpha}$, $\hat{\beta}$, $\hat{\theta}$, $\hat{\lambda}$, $\hat{\sigma}_c^2$, and $\hat{\sigma}_s^2$ represent the solution to equations (A-23) and (A-29), then the invariance

theorem for maximum likelihood estimation can be used to obtain

$$\hat{\rho} = \frac{(\hat{\lambda})^2 \hat{\sigma}_s^2}{\hat{\sigma}_c^2 + (\hat{\lambda})^2 \hat{\sigma}_s^2} \quad (\text{A-29})$$

as the maximum likelihood estimate of ρ .

The equations (A-23) through (A-29) are nonlinear but can be solved using numerical techniques. Newton's Method was used to solve the equations for this report; i.e., if $u^{(k)}$ is an estimate of the solution vector $u = (\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}, \hat{\sigma}_c^2, \hat{\sigma}_s^2)$ at the k th step, then

$$u^{(k+1)} = u^{(k)} - F^{-1}_f(u^{(k)}) \quad (\text{A-30})$$

where $f(u^{(k)}) = (f_1, \dots, f_6)^T$ is the vector of the left sides of equations (A-23) through (A-29) evaluated at $u^{(k)}$ and $F = (F_{ij}) = \frac{\partial f_i}{\partial u_j}$.

In practice, it was slightly more simple to use the parameter transformations

$$r = \frac{\sigma_s^2}{\lambda^2 \sigma_s^2 + \sigma_c^2} \quad (\text{A-31})$$

and
$$s = \lambda^2 \sigma_s^2 + \sigma_c^2 \quad (\text{A-32})$$

and solve for $\alpha, \beta, \theta, \lambda, r,$ and s . Again, the invariance theorem can be used to give

$$\hat{\rho} = \hat{\lambda}^2 \hat{r}$$

A.3.1.5.4 Accuracy of $\hat{\rho}$

Since $\hat{\rho}$ is an extremely complicated function of the data, it is impossible to write down the variance of $\hat{\rho}$ for finite sample sizes m and n . However, the asymptotic variance of $\hat{\rho}$ can be estimated using the information matrix; i.e., if

$$V = E \left\{ \frac{-\partial^2 \log L}{\partial u_i \partial u_j} \right\}$$

and $g(u) = g(\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}, \hat{\sigma}_c^2, \hat{\sigma}_s^2)$ is a differentiable function of the parameter vector u , then the variance of $g(u)$ is asymptotic to

$$[g'(u)]^T V^{-1} g'(u)$$

where $g'(u) = \left(\frac{\partial g}{\partial u_1}, \dots, \frac{\partial g}{\partial u_6} \right)^T$. (A-33)

Thus, in our case, $g(u) = \frac{\lambda^2 \sigma_s^2}{\lambda^2 \sigma_s^2 + \sigma_c^2}$

$$g'(u) = \begin{bmatrix} 0, 0, 0, 2\lambda \sigma_s^2 \sigma_c^2 (\lambda^2 \sigma_s^2 + \sigma_c^2)^{-2}, -\lambda^2 \sigma_s^2 (\lambda^2 \sigma_s^2 + \sigma_c^2)^{-2}, \\ -\frac{\lambda^2 \sigma_c^2}{(\sigma_c^2 + \lambda^2 \sigma_s^2)^2} \end{bmatrix} \quad (A-34)$$

To estimate V , the observations $\{X_i\}$, $\{Y_i\}$, and $\{Z_i\}$ and the estimated parameters $(\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}, \hat{\sigma}_c^2, \text{ and } \hat{\sigma}_s^2)$ were substituted into the matrix $H = (h_{ij}) = \frac{\partial^2 \log L}{\partial u_i \partial u_j}$. Then equation (A-33) was used to obtain an approximate variance for $\hat{\rho}$.

A.3.2 YIELD

This section contains a description of the methods used to predict yields (section A.3.2.1) and to estimate yield prediction error (section A.3.2.2). In Phase II no estimate of yield bias was made.

A.3.2.1 Yield Prediction

Most of the yield predictions made in LACIE are provided by the Center for Climatic and Environmental Assessment (CCEA) of NOAA. They are produced from multiple linear regression yield models* developed on historical weather and yield data. Usually these models cover a state but in some cases they cover part of a state or part of two states and in some cases they overlap.

In a given state there is either one yield stratum or two. In the first case the state yield prediction is that given by the CCEA model. In the second case the state yield prediction is given by:

$$Y = P/A \quad (A-35)$$

where P is the production estimate (section A.3.3.1) and A is the acreage estimate (section A.3.1.2) for the state. The yield prediction at the region or country level is also obtained from equation (A-35), with P and A in that case being the production and acreage estimates at the corresponding level.

A.3.2.2 Estimation of the Yield Prediction Error

CCEA provides estimates of the yield prediction error at the stratum level. In the CAS Requirements Document it is shown that at the state, region, or country levels the estimate of the squared yield prediction error for a given area (state, region, or country) is

$$U^2 = \bar{Y}^2 \left[\frac{S^2}{P^2} + \frac{V^2}{A^2} - 2 \frac{\sum Y_i V_i^2}{P A} \right] \quad (A-36)$$

*Wheat Yield Models for the United States (LACIE 00431), National Aeronautics and Space Administration, Johnson Space Center, Houston, Texas, June 1975.

where

s^2 = estimated squared prediction error of the production estimate P for the area

v^2 = estimated variance of the acreage estimate A for the area

y_i = yield estimate for the *i*th pseudo zone in the area

v_i^2 = estimated variance of the acreage estimate for the *i*th pseudo zone in the area

In the case where there is only one yield stratum for a state, the yield prediction error for the state is given directly by the CCEA model.

A.3.3 PRODUCTION

This section contains descriptions of the methods used to do the following:

- a. Estimate wheat production (section A.3.3.1).
- b. Estimate the variance in the wheat production estimate (section A.3.3.2).
- c. Estimate the bias in the wheat production estimate (section A.3.3.3).
- d. Evaluate whether LACIE is satisfying the 90/90 criterion (section A.3.3.4).
- e. Determine the effect of errors in acreage, yield, sampling, and classification on the production variance (section A.3.3.5).

A.3.3.1 Production Estimation

At the CRD level the production estimate is obtained by multiplying the area estimate and the yield prediction for the CRD. The area estimate is made for the CRD itself but the yield prediction is made for a group of CRD's in a state (section A.3.2.1).

The production estimates for the state and higher levels are obtained by simply adding the estimates for all the CRD's in the area.

A.3.3.2 Production Variance Estimation

Since the production estimate is the product of an acreage estimate and a yield prediction, the measure of variability in the estimate should properly be called the production prediction error. However, in this report, this quantity will be called the production variance.

Since the yield predictions are made for a group of CRD's it is not possible to obtain independent production variance estimates at the CRD level. Hence, the estimates of production variance are made only at the state and higher levels.

To estimate the production variance for a state it is assumed that the yield strata do not cross a CRD. This seems a reasonable assumption and is expected to hold in almost all cases. Another assumption is that the yield strata are nonoverlapping. However, this does not hold for the North Dakota and Minnesota yield strata since CRD's 30 and 60 in North Dakota are a part of both yield strata. Similarly, there is an overlap in Nebraska and South Dakota where CRD 10 of Nebraska is common to both yield strata, and in Oklahoma and Texas where CRD 10 of Oklahoma is common to both Oklahoma yield stratum and the Texas Panhandle yield stratum. In Phase II, any such overlapping is ignored and production variance estimates are considered approximate.

Regarding the number of yield strata in a state, in Phase II only two cases occurred in the USGP, namely (1) a single yield model in a state, and (2) two yield models in a state.

Single Yield Model in a State

In the CAS Requirements Document it is shown that when there is only one yield model in a state, an estimate of the production variance is given by

$$S^2 = V^2Y^2 + U^2A^2 - V^2U^2 \quad (\text{A-37})$$

where

P = state production estimate

Y = yield prediction for the state from the state yield model

U^2 = the estimated squared yield prediction error for the state

A = the state acreage estimate obtained by summing the acreage estimates for the CRD's in the state

V^2 = the estimated state acreage variance

Two Yield Models in a State*

When there are two yield models in a state, the state is divided into two pseudo zones corresponding to the intersections of the two yield strata with the acreage strata in the state. Let G_1 and G_2 denote the pseudo zones associated with yield strata 1 and 2 having yield estimates Y_1 and Y_2 respectively. The acreage estimates A_1 and A_2 for G_1 and G_2 are given by

$$A_t = \sum_{j \in G_t} A_j, \quad t = 1, 2 \quad (\text{A-38})$$

where A_j is the acreage estimate for the j th CRD in the state.

*This discussion is only for the nonoverlapping yield strata and does not address the problem of a mixed wheat zone.

It is shown in the CAS Requirements Document that an estimate of the production variance is given by

$$s^2 = \sum_{t=1}^2 \left(v_t^2 Y_t^2 + U_t^2 A_t^2 - v_t^2 U_t^2 \right) + 2Y_1 Y_2 \sum_{j \in G_1} \sum_{k \in G_2} \psi_{jk} \quad (\text{A-39})$$

where U_t^2 is the estimated squared prediction error of Y_t , ψ_{jk} is the estimated covariance between A_j and A_k and v_t^2 is the estimated variance of the acreage estimate A_t given by

$$v_t^2 = \sum_{j \in G_t} v_j^2 + 2 \sum_{j \in G_t} \sum_{k \in G_t} \psi_{jk} \quad (\text{A-40})$$

Here v_j^2 is the acreage variance estimate for the j th CRD. For more details on these calculations see the CAS Requirements Document.

The production variance for a region or country is estimated by adding the estimated production variances for the states in the region or country. This, however, ignores the covariances between the state production estimates caused by some yield strata crossing the state boundaries, as mentioned earlier. This problem is being corrected during LACIE Phase III.

The procedure for estimating the production variance in a mixed wheat area is the same for spring wheat, winter wheat, and total wheat. However, in the case of total wheat, the yield prediction and yield prediction error required for this are obtained by combining the corresponding quantities for spring and winter wheat with relative weights based on the previous year's SRS spring and winter wheat acreages.

A.3.3.3 Production Bias Estimation

The production bias at the state level is given by

$$\begin{aligned}
 B_{P_i} &= E(\hat{P}_i - P_i) \\
 &= E(\hat{P}_i) - P_i \\
 &= E(\hat{A}_i \hat{Y}_i) - A_i Y_i
 \end{aligned}
 \tag{A-41}$$

where A_i , Y_i , and P_i are respectively the true values of the acreage, yield, and production for the N^{th} state in question, and \hat{A}_i , \hat{Y}_i , and \hat{P}_i are the corresponding estimates for these quantities. Assuming \hat{A}_i and \hat{Y}_i are independent, one obtains

$$B_{P_i} = E(\hat{A}_i)E(\hat{Y}_i) - A_i Y_i \tag{A-42}$$

If one further assumes that Y_i is unbiased, then $E(\hat{Y}_i) = Y_i$, and

$$\begin{aligned}
 B_{P_i} &= Y_i [E(\hat{A}_i) - A_i] \\
 &= Y_i B_{A_i}
 \end{aligned}
 \tag{A-43}$$

where B_{A_i} is the acreage bias for the i^{th} state. The quantities Y_i and B_{A_i} are unknown, but an estimate, \hat{B}_{P_i} for B_{P_i} can be obtained by using the estimates for Y_i and B_{A_i} described in sections A.3.2.1 and A.3.1.4, respectively. Thus,

$$\hat{B}_{P_i} = \hat{Y}_i \hat{B}_{A_i} \tag{A-44}$$

The variance of \hat{B}_{P_i} is given by

$$\text{Var}(\hat{B}_{P_i}) = \hat{Y}_i^2 \text{Var}(\hat{B}_{A_i}) + B_{A_i}^2 \text{Var}(\hat{Y}_i) + \text{Var}(\hat{B}_{A_i}) \text{Var}(\hat{Y}_i)$$

and estimated by

$$\hat{\text{var}}(\hat{B}_{P_i}) = \hat{Y}_i^2 \hat{\text{var}}(\hat{B}_{A_i}) + \hat{B}_{A_i}^2 \hat{\text{var}}(\hat{Y}_i) - \hat{\text{var}}(\hat{B}_{A_i}) \hat{\text{var}}(\hat{Y}_i)$$

For the nine-state level, the production bias estimate \hat{B}_P is simply given by $\hat{B}_P = \Sigma \hat{B}_{P_i} = \Sigma \hat{Y}_i \hat{B}_{A_i}$ and the estimate of its variance is $\Sigma \hat{V}ar(\hat{B}_{P_i})$. The relative bias of the production estimate $R(\hat{B}_P)$ is estimated by expressing the production bias as a percentage of the LACIE production estimate, i.e., by

$$R(\hat{B}_P) = \frac{\Sigma \hat{Y}_i \hat{B}_{A_i}}{\Sigma \hat{A}_i \hat{Y}_i} \times 100 \quad (A-45)$$

A.3.3.4 Evaluating the 90/90 Criterion

Let \hat{P} be the LACIE estimate of wheat production for the region or country, and let P be the true wheat production of the same region or country. The accuracy goal of the LACIE is a 90/90 at-harvest criterion for wheat production, which is given by the following probability statement.

$$\Pr \left[|\hat{P} - P| \leq 0.1P \right] \geq 0.90 \quad (A-46)$$

This states that the accuracy goal is for the LACIE estimate of wheat production to be within 10 percent of the true wheat production with a probability of at least 0.9.

It is assumed that the LACIE estimate, \hat{P} , is normally distributed with mean $P + B$ and variance $\sigma_{\hat{P}}^2$, where

$$B = E(\hat{P}) - P$$

Under this assumption, equation (A-46) may be written as

$$\Pr \left[\frac{-0.1 - 0.9 \frac{B}{P+B}}{CV(\hat{P})} \leq Z \leq \frac{0.1 - 1.1 \frac{B}{P+B}}{CV(\hat{P})} \right] \geq 0.90 \quad (A-47)$$

where $Z = \frac{P - (P+B)}{\sigma_{\hat{P}}}$ follows the standard normal distribution,

$N(0,1)$, and $CV(\hat{P})$ is the coefficient of variation of \hat{P} defined by

$$CV(\hat{P}) = \frac{\sigma_{\hat{P}}}{E(\hat{P})} = \frac{\sigma_{\hat{P}}}{P+B} \quad (A-48)$$

The term $\frac{B}{P+B}$ is called the relative bias of \hat{P} and is given by

$$\frac{E(\hat{P}) - P}{E(\hat{P})} = \frac{B}{P+B}$$

It follows that the accuracy goal of LACIE is attained if

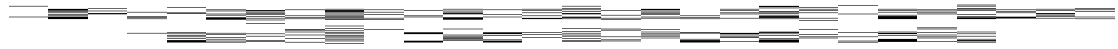
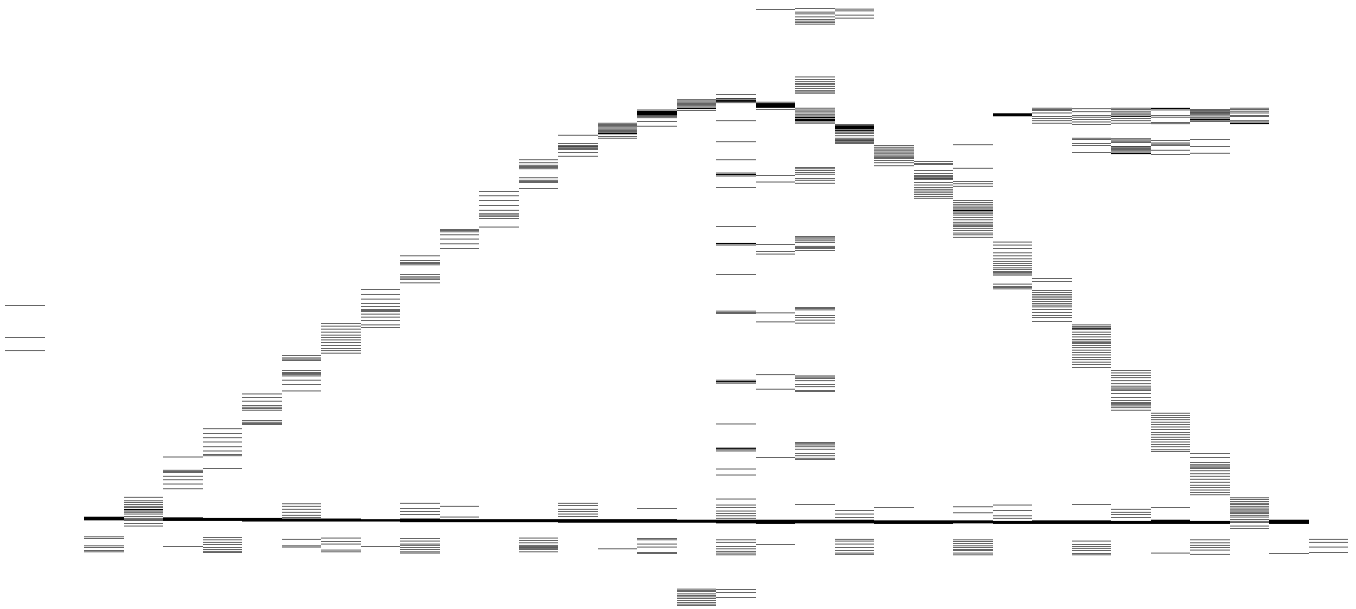
$$\Phi \left[\frac{0.1 - 1.1 \frac{B}{P+B}}{CV(\hat{P})} \right] - \Phi \left[\frac{-0.1 - 0.9 \frac{B}{P+B}}{CV(\hat{P})} \right] \geq 0.90 \quad (A-49)$$

where Φ represents the cumulative standard normal distribution. Figure A-1 is a plot of the relative bias versus the coefficient of variation to the LACIE wheat production estimate necessary to satisfy equation (A-49), replacing the inequality sign with an equal sign.

Inference as to whether the LACIE accuracy goal has been met is made by estimating $\frac{B}{P+B}$ and $CV(\hat{P})$ and then ascertaining whether equation (A-48) has been satisfied. Although the LACIE accuracy goal applies to the at-harvest estimate of wheat production, discussion of the 90/90 criterion is made in each interim report as applied to the region for which the LACIE estimates of wheat production are available.

A.3.3.5 Effect of Errors in Acreage, Yield, Sampling, and Classification on the Production Variance

The production variance consists of two major error components: acreage and yield. The acreage error may be further subdivided into sampling and classification errors. The effect of a particular error is determined by the reduction in the production variance estimate when the error is omitted from the calculation of



that estimate. These determinations are carried out at the state and higher levels.

At the state level there are two cases to consider: (1) one yield model in the state, and (2) two yield models in the state. When there is one yield model in a state the production variance with all the error components included is given by equation (A-37).

In order to determine the variance without a given error term, equation (A-37) must be re-derived with that term omitted. Let S_A^2 , S_Y^2 , S_S^2 and S_C^2 be the state production variances without acreage, yield, sampling, and classification errors respectively. Using the above-mentioned procedure, one obtains the following expressions for these quantities:

$$S_A^2 = U^2(A^2 - V^2) \quad (A-50)$$

$$S_Y^2 = V^2(Y^2 - U^2) \quad (A-51)$$

$$S_S^2 = (1 - \hat{\rho})V^2(Y^2 - U^2) + U^2A^2 \quad (A-52)$$

$$S_C^2 = \hat{\rho} V^2(Y^2 - U^2) + U^2A^2 \quad (A-53)$$

Here U, V, Y and A are as defined in section A.3.3.2 and $\hat{\rho}$ is defined by equation (A-29). It should be noted that the expression for the production variance without acreage error, equation (A-50), is not the expression that would be obtained by simply setting the acreage variance, V, equal to zero in equation (A-37). A similar observation applies to equation (A-15).

When there are two yield models in a state the production variance with all the error components included is given by equation (A-39). In this case the estimates for S_A^2 , S_Y^2 , S_S^2 and S_C^2 are given by

$$S_A^2 = \sum_{t=1}^2 U_t^2(A_t^2 - V_t^2) \quad (A-54)$$

$$S_Y^2 = \sum_{t=1}^2 V_t^2 (Y_t^2 - U_t^2) + 2Y_i Y_2 \sum_{j \in G_1} \sum_{k \in G_2} \psi_{jk} \quad (A-55)$$

$$S_S^2 = \sum_{t=1}^2 \left[(1 - \hat{\rho}) V_t^2 (Y_t^2 - U_t^2) + U_t^2 A_t^2 \right] + 2Y_i Y_2 \sum_{j \in G_1} \sum_{k \in G_2} \psi_{jk} \quad (A-56)$$

$$S_C^2 = \sum_{t=1}^2 \left[\hat{\rho} V_t^2 (Y_t^2 - U_t^2) + U_t^2 A_t^2 \right] + 2Y_i Y_2 \sum_{j \in G_1} \sum_{k \in G_2} \psi_{jk} \quad (A-57)$$

Here U_t , V_t , Y_t and A_t are as defined in section A.3.3.2 and $\hat{\rho}$ is defined by equation (A-29).

In order to calculate the quantities corresponding to S_A^2 , S_Y^2 , S_S^2 , and S_C^2 at the regional and country levels, it is assumed that the state production estimates are independent. The corresponding quantities are then obtained by adding the estimates for the states in the area.

In Phase II the necessary software was not available to perform the calculations using equations (A-54) through (A-57). Therefore, the results in this report were obtained using equations (A-50) through (A-53).

APPENDIX B
PHASE II BLIND SITE DATA

APPENDIX B

PHASE II BLIND SITE DATA

The following tables give the Phase II blind site data. The headings are read from top to bottom and the following quantities are given:

State name

State code

CRD number

Segment number

Acquisition date

CAMS code

Biowindow

CAMS proportion estimate

Crop W = winter wheat

B = winter small grains

K = small grains

Wheat classification accuracy

Non-wheat classification accuracy

Small grains proportion (percent) - includes wheat

Wheat proportion (percent)

Other small grains proportion (percent) - i.e., other than wheat

Abandoned wheat proportion (percent)

Abandoned other grains (percent)

1969 agricultural census percent wheat for the county containing
the segment code

AI code

Estimate of biostage (on the Robertson scale)

TABLE B-1.- Concluded.

(b) Winter Wheat

S	S				B																				
TN	TC			D	CC	IO	C	C	W		SG	W	OSG	W	OG										
AA	AO	C	S	AA	AO	W	AE	R	HCA	NCA	MR	H	TMR	HA	TRA	AE									
TM	TD	RN	EN	CT	MD	I	MS	D	ELC	WLC	AAP	EP	HAAP	FBP	HABP	9RSP	C								
EE	EE	DO	GO	QE	SE	N	ST	P	TFY	TFY	LNT	TT	RLNT	TNT	RNNT	6IUC	AD								
																9CST	IE								
MONT	30	50	1742	1066	20		15.5	B	94.4	96.9	34.1	25.1	9.0	00.0	00.0	7.4	73	2.4							
MONT	30	50	1742	1376	30	1	14.7	W	99.4	98.5	34.1	25.1	9.0	00.0	00.0	7.4	37	3.0							
MONT	30	50	1743		36	1	.4	W			.5	.5	00.0	00.0	00.0	7.4	30	3.0							
MONT	30	80	1550	1536	34	2	.4	W			.4	.4	00.0	00.0	00.0	1.6	33	3.8							
MONT	30	30	1534	1506	30	2	12.5	W	96.5	90.6	9.6	6.8	2.3	00.0	00.0	7.6	40	3.6							
MONT	30	30	1536	1706	36	2	1.8	K			3.2	2.5	.8	00.0	00.0	1.1	32	3.6							
MONT	30	30	1536	2246	36	4	1.4	K			3.2	2.5	.8	00.0	00.0	1.1	32	6.0							

APPENDIX C
PHASE I INTENSIVE TEST SITES

APPENDIX C

PHASE I INTENSIVE TEST SITES

To accomplish the objectives of accuracy assessment, ground truth, aircraft photographs, and Landsat multispectral scanner imagery were gathered from 29 intensive test sites. A complete list of these sites and their locations is given in table C-1. The Landsat acquisitions obtained for each site are shown in table C-2. Because of factors such as atmospheric effects and data dropout, six of the sites did not have enough acquisitions to satisfy the CAMS rework criteria (page 3-5 of this report).



TABLE C-2.- INTENSIVE TEST SITE ACQUISITIONS LISTED BY
 BIOPHASE ACCORDING TO DAY OF ACQUISITION, 1975

Segment	Biophase			
	1	2	3	4
1687	133		205	
1960	291		150	
1961	291			169
1962	324	131		
1963	289	131		
^a 1964	290			
1965	155	191		
^a 1966				
1967	137	191		227
1968	143	180	216	
1969	161	179	215	233
1970	142	179		233
^a 1971	142			
1972	268			218
1973	268		201	218
1974	268		182	218
^b 1975	159	178	195	213
1976	299	177	195	213
1977	299		196	214
1978	291		133	
1979	291		133	
1980	291		133	
^b 1981	105			176
1982	299	140		
1983	281	141		
^a 1984		195		
^a 1985				
1986	150	169	187	
^a 1987				

^aSegments for which the acquisitions do not satisfy the CAMS rework criteria.

^bSegments moved to coincide with ground truth and thus reordered.

APPENDIX D

This appendix presents the results of aggregating ground-observed wheat proportions for the blind sites in the USGP (table D-1). These aggregated area estimates contain only sampling and Group III errors but no classification errors. A statistical test (described in section A.2) shows that at the 10-percent level there is a significant difference between the blind site aggregated and the December 1976 USDA/SRS area estimates only for the state of Colorado. That is, if the LACIE area estimate had no classification error, it would agree very well with the USDA/SRS estimate for every state in the USGP except Colorado.

TABLE D-1.— RESULTS OF AGGREGATING GROUND-OBSERVED WHEAT PROPORTIONS FOR THE BLIND SITES IN THE USGP

State	Blind sites	Blind sites aggregated wheat	Blind site CV, %	December 1976 SRS estimate
Winter wheat				
Colorado	13	3 719	24.4	2 200
Kansas	35	12 163	5.5	11 300
Nebraska	18	3 187	15.2	2 950
Oklahoma	19	5 294	20.6	6 300
Texas	18	4 930	21.4	4 700
USSGP	103	29 293	6.7	27 450
Montana	11	2 889	73.8	3 080
S. Dakota	5	1 536	45.8	970
MW states	16	4 425	50.7	4 050
USGP-7	119	33 718	8.8	31 500
Spring wheat				
Minnesota	5	3 689	17.1	3 893
Montana	7	2 056	28.8	2 335
N. Dakota	13	11 541	14.2	11 520
S. Dakota	6	2 677	19.5	2 020
USGP-4	31	19 963	9.6	19 768
Total wheat				
USNGP	47	24 388	12.1	23 818
USGP-9	150	53 681	6.7	51 268